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Low Order Controller Synthesis for Interval System

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Abstract-This brief considers the problem of controller synthesis for interval system using the method of order reduction technique and model matching concept. The order reduction is accomplished by using the combined benefits of approximate generalized time moment(AGTM)matching and optimization procedure. Luus –Jaakola(LJ) optimization procedure is used for minimizing the performance index subject to the constraints. Using this method, controller of any order can be obtained with guaranteed stability and performance. The illustrated example demonstrates the effectiveness of the projected method.

Keywords—**Interval systems, approximate generalized time moment matching, model order reduction, stability.**

I. INTRODUCTION

In practical application, due to hardware or computational limitations, there is a need to control complex systems using low order controllers. Hence, proportional-integral – derivative(PID) controllers and lead/lag compensators are the most popular types of control in industrial applications, due to their simplicity in implementation and maintenance, and their ability to effectively control the process. In modern control dynamics controller design frequently results in higher order controllers. Controller order reduction is very important issue in many control applications. Recently, many parametric approaches [i-iii] have been explored and developments have been made in the synthesis of low order controllers. Simple linear controllers are normally preferred over complex linear controllers for linear time invariant plants. A system with the high order controller poses difficult in its analysis, synthesis or identification . An obvious method of dealing with such system with higher order controller is to approximate them by low-order controller which reflect the characteristics of original system. It is a desireable to have methods available for designing low order controllers for high order plants. Such methods can broadly be classified into indirect method and direct method , There are two approaches in the indirect method. In the first approach, the order of the plant is reduced and then a controller is designed for the reduced order plant. In the second approach, a controller is designed for the full order plant and then reduced order controller is obtained.

In direct approach, a quadratic optimization problem is posed with an order constraint and, naturally a closed-loop stability constraint. Then there are two main issues to be considered. The first is that of providing a satisfactory numerical procedure for executing the optimization. This is far from being a trivial task; no procedures are yet available in commercial control system software design packages. This leads to the second issue, which relates to utility of the whole approach for achieving closed-loop design goals apart from minimization of a performance index.

Even though significant progress has been made recently in the area of interval control, this is still an open problem for which a very few results are available. In literature it is pointed out that a significant deficiency of control theory at the present time is the lack of non-conservative design methods to achieve robustness under parameter uncertainty. Most of the existing results in the area of parametric robust control are analysis results. In this paper the problem of low order controller synthesis is accomplished by order reduction technique and model matching concept.

II. METHOD OF MODEL ORDER REDUCTION

Some of the model order reduction techniques for fixed coefficient system in frequency domain are applicable to interval systems also [vi-xiv]. In this paper the problem of model order reduction for interval system is divided into two phases. In the first phase, nominal transfer function is obtained from given higher order interval transfer function. The order of nominal transfer function is reduced using AGTM matching method in which the optimum selection of expansion points is carried out using the LJ optimization procedure. In the second phase, the optimization procedure is done for obtaining the interval parameters.

A. FIRST PHASE

Consider a stable linear time invariant single input single output interval system described by the higher order nominal transfer function,

$$G(s)=\frac{c_q s^q + c_{q-1}s^{q-1}+c_{q-2}s^{q-2}+..........+c_1 s+c_0}{s^p + d_{p-1}s^{p-1}+d_{p-2}s^{p-2}+............+d_1 s+d_0} \quad (1)$$

where c_i , $0 \le i \le q$ and d_i, $(0 \le i \le p-1)$ are known scalar constants ,q and p are order of numerator and denominator of higher order transfer function.

Let the reduced order model be of the form

$$R(s)=\frac{b_m s^m + b_{m-1}s^{m-1}+b_{m-2}s^{m-2}+.........+b_1 s+b_0}{s^n + a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+...........+a_1 s+a_0.} \quad (2)$$

where b_i, $(0 \leq i \leq m)$ and a_i, $(0 \leq i \leq n-1)$ are unknown scalar constants, m and n are order of numerator and denominator of low order transfer function.

In this paper, a method is proposed to obtain reduced order nominal model in the form of equation (2) such that the reduced order model retains important characteristics of the original system and approximates its response as closely as possible for the same type of inputs.

Steady state value of the higher order model,

$$g(\infty) = \frac{c_0}{d_0}$$

Steady state value of the reduced order model,

$$r(\infty) = \frac{b_0}{a_0}$$

Equating the two steady state values,

$g(\infty) = r(\infty)$, expression for b_0 is obtained as

$$b_0 = a_0 g(\infty) \qquad (3)$$

Hence the number of unknown parameters to be found out is reduced by one.

Matching the approximate generalized time moments of the higher order model, $G(s)$ with that of the reduced order model, $R(s)$ at the

expansion points s =s_i , i=1,2,3…………(m+n).

$$G(s_i) = R(s_i) \qquad (4)$$

G(s), for a particular value of frequency, s =s_i can be written as V_i

$$G(s)\big|_{s = s_i} = V_i \qquad (5)$$

Equating equation (4) and (5)

$$V(s_i^{\,n} + S_a X_1 + a_0) = S_b X_2 + b_0 \qquad (6)$$

where

$$X_1 = \begin{bmatrix} a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_1 \end{bmatrix}^T$$
$$X_2 = \begin{bmatrix} b_m & b_{m-1} & b_{m-2} & \cdots & b_1 \end{bmatrix}^T$$
$$S_a^{(i)} = \begin{bmatrix} s_i^{n-1} & s_i^{n-2} & s_i^{n-3} & \cdots & s_i \end{bmatrix}$$
$$S_b^{(i)} = \begin{bmatrix} s_i^{m} & s_i^{n-2} & s_i^{n-3} & \cdots & s_i \end{bmatrix}$$

(7)

Equation (6) can be written as

$$S_b^{(i)} X_2 + a_0 (g(\infty) - V_i) - V_i S_a^{(i)} X_1 = V_i s_i^{\,n} \qquad (8)$$

Equation (8) can be written as

$$\begin{bmatrix} S_b^{(i)} & (g(\infty) - V_i) & -V_i S_a^{(i)} \end{bmatrix} \begin{bmatrix} X_2 \\ a_0 \\ X_1 \end{bmatrix} = V_i s_i^{\,n} \qquad (9)$$

For a particular value of 's' or for a particular frequency, LHS of equation(9) is multiplication of unknown column matrix with a row matrix and RHS is a single value. Hence for another value of 's', another set is obtained. Thus different rows are obtained for forming matrix A and B. Thus equation (9) can be written as

$$AX = B \qquad (10)$$

where $X = \begin{bmatrix} X_2 \\ a_0 \\ X_1 \end{bmatrix}$

(11)

In the equation (10) the elements of A and B are known values. Using MATLAB programming, the elements of unknown matrix, X can be found out. The unknown parameter b_0 is obtained from the equation (3). Thus the unknown coefficients of the reduced order nominal model can be obtained.

B. SECOND PHASE

In the case of interval transfer function, the first step is to obtain the higher order nominal transfer function of the given higher order interval transfer function. The algorithm explained above is used for getting the reduced order nominal model in the form,

$$R(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \ldots\ldots \ldots\ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots\ldots \ldots\ldots + a_1 s + a_0}$$

In the paper, the problem of choosing the best e_i has been cast as a constrained optimization problem which is solved by Luus-Jaakola (LJ) random-search method.

The optimization problem is given as follows

Find e_i ,$i=0$ *to* $m+n+1$ so as to:

Minimize $j = \int \left(Y_m(t) - Y_c(t) \right)^2 dt$

where $Y_m(t)$ is the step response of the higher order model ,and $Y_c(t)$ is the response of the low order model.

Subject to the constraints:

$\{real[roots(K_i)]\} < 0$, i=1 to 4, where

K_i are Kharitonov polynomials[v] of reduced interval transfer function.

The desired reduced order interval model is in the form

$$G_I(s) = \frac{[b_m - e_m, b_m + e_m]s^m + [b_{m-1} - e_{m-1}, b_{m-1} + e_{m-1}]s^{m-1} + \ldots + [b_1 - e_1, b_1 + e_1]s + [b_0 - e_0, b_0 + e_0]}{s^n + [a_{n-1} - e_{m+n+1}, a_{n-1} + e_{m+n+1}] s^{n-1} + [a_{n-2} - e_{m+n}, a_{n-2} + e_{m+n}]s^{n-2} + \ldots + [a_1 - e_{m+2}, a_1 + e_{m+2}]s + [a_0 - e_{m+1}, a_0 + e_{m+1}]}$$

III. LOW ORDER CONTROLLER DESIGN

Several approaches are existing for the design of low order controller for linear time invariant single input single output systems with interval parameters. The proposed method is for designing a stabilizing controller for the stable higher order system while preserving the closed loop performance. The indirect approach is adopted for controller design which helps to match the desired closed loop response with the designed closed loop response. But the higher order controller is designed by model matching concept. After that the order of controller is reduced using the method of model order reduction technique explained in the above sections.

The following steps describes the method used in this paper for designing the low order controller for interval system.

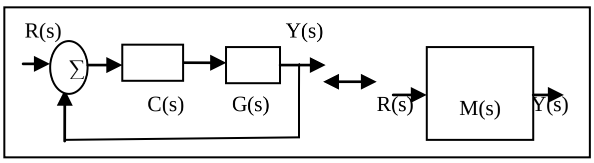

Fig.1: General closed loop configuration with unity feedback

Step 1 .In the fig.1 , G(s) , C(s) and M(s) are the high order interval plant, controller and desired closed loop transfer function respectively. G(s) and M(s) are known. The expected improvement in the system performance is depicted in the desired closed loop transfer function. Equating the desired closed loop transfer function, M(s) to the closed loop transfer function of the system, the equation for higher order interval controller ,C_{HI} is obtained as

$$C_{HI} = \frac{M(s)}{[1 - M(s)]G(s)}$$

Step 2. The higher order nominal controller is obtained from the higher order interval controller.

Step 3. The model reduction method proposed in the previous sections can be used for controller reduction. The fundamental difference is, in place of higher order plant, higher order controller is used. In the optimization procedure , closed loop response is considered instead of open loop response which is used in model reduction.

The method directly yields a low-order implementable controller of pre-specified structure that ensures stability and captures the essential characteristics of the desired model M*(s)*, based on AGTM matching, LJ optimization procedure and the concept of model matching.

IV. SIMULATION RESULT

The developed method is applied for designing interval controller for several interval systems. It is clear from the illustrated examples that the proposed method is efficient for designing interval controller which will guarantee the stability and performance matching.

For simulation, interval transfer function is given as $G_I(s)$ and desired closed loop interval transfer function is given as $M_I(s)$. Higher order interval controller is termed as $C_{HI,}$ and M_{NOM}, is the nominal values of $M_I(s)_I$.

The low order interval controller is obtained as C_{LI} and the integral square error between step response of desired closed loop transfer function and designed closed loop transfer function is obtained as J. The designed closed loop interval transfer function is obtained as $M_{ID}(s)$ and nominal transfer function of $M_{ID}(s)$ is termed as $M_{DNOM.}$

$$G_I(s) = \frac{[1,1]s^2 + [1,2]s + [1,2]}{[1\ ,1]s^4 + [3,4]s^3 + [4,4]s^2 + [5,8]s + [1,1]}$$

The fourth order interval transfer function of a plant is taken from [xv]. The desired closed loop transfer function is generalized as follows

$$M_I(s) = \frac{\omega_n^2}{[1,1]s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\text{For } \omega_n = [1, 2]; \quad \zeta = [4, 6]$$

$$M_I(s) = \frac{[1,4]}{[1,1]s^2 + [8, 24]s + [1, 4]}$$

The higher order interval controller is designed from the desired closed loop transfer function and transfer function of the interval plant.

$$C_{HI} = \frac{[1,4]s^4 + [3,16]s^3 + [4,16]s^2 + [5,32]s + [1,4]}{[1,1]s^4 + [9,26]s^3 + [6,53]s^2 + [2,54]s + [-6,6]}$$

After obtaining the higher order nominal interval transfer function, the order is reduced using the above described method of model reduction.

The following are the results obtained from simulation.

$$C_{LI} = \frac{[0.3323, 0.5741]s + [0.0949, 0.1153]}{[1,1]s + [-0.0055, 0.0089]}$$

The step response of the desired closed loop transfer function with that of designed closed loop transfer function is compared. The Fig: 2. shows the comparison

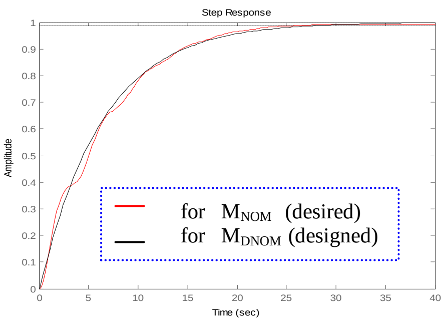

Fig. 2: Comparison of step responses of M_{NOM} and M_{DNOM}

From the illustrated examples, it is clear that the proposed method can guarantee the performance matching. The step response of desired closed loop transfer function matches very closely with that of designed closed loop transfer function. At the same time stability is also preserved. Controller of arbitrary order can be designed using the developed method. For using this method, there is no restriction to the order of numerator and denominator of the higher order controller.

V. CONCLUSION

In this paper, a new method of controller synthesis for interval systems is presented. The method is mathematically simple, easy for computation and can be used for designing controller also.

There are several ways of obtaining a low-order controller for a high-order system. One line of thinking is to first obtain a very accurate higher order model and then apply reduction techniques to this model or to a higher order controller computed from that model. There is an extensive literature on this subject. One of the important theoretical messages of this literature is that, if the ultimate objective is the low-order controller (rather than the low-order model), then it is essential that the closed-loop performance objective be incorporated in the reduction technique. In this paper closed loop performance is considered and compared the response of designed closed loop with that of desired closed loop response. The responses are comparable and matching closely. Using Kharitonov's theorem, sufficient and necessary conditions are derived for the existence of a stabilizing controller for a given interval system. The work can be extended for multi input multi output interval systems also.

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