International Journal of Engineering Research
Volume No.3, Issue No.2, pp : 79-82
(ISSN:2319-6890)(online),2347-5013(print)
01 Feb. 2014

On Flow Past A Parabolic Started Infinite Vertical Plate With Variable Temperature And Uniform Mass Diffusion

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ABSTRACT : An exact solution of unsteady flow past a parabolic motion of an infinite vertical plate with variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations are solved using Laplace transform in which the velocity increases with increasing values of thermal Grashof number or mass Grashof number.

Key words - parabolic, vertical plate, variable, uniform, heat and mass transfer.

INTRODUCTION

In many process industries, such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, molten plastics, pulps, paper industry, textile industry, the cooling of threads or sheets of some polymer materials is of importance in the industrial applications.

Natural convection on flow past a parabolic infinite vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar(1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo(1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991). The effect of a transverse magnetic field on unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate was analyzed by Agrawal et al (1999). The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite vertical plate subjected to parabolic motion with variable temperature and mass diffusion, in the presence of transverse magnetic field. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable mass diffusion has been considered. The x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is started with a velocity $u = u_0 . t'^2$ in its own plane against gravitational field and the temperature from the plate raised linearly with time t and the mass is diffused from the plate to the fluid uniformly. Since the plate is infinite in length all the terms in the governing equations will be independent of x and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_P \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$u = 0,\ T = T_\infty,\ C' = C'_\infty \ \text{ for all } \ y, t' \le 0$$

$t' > 0: u = u_0 t'^2,\ T = T_\infty + (T_w - T_\infty)At',\ C' = C'_w$ at $y = 0$ (4)

$$u \to 0 \quad T \to T_\infty,\ C' \to C'_\infty \quad \text{as} \quad y \to \infty$$

Where $A = \frac{u_0^{\,2}}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3},\quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t',\quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T - T_\infty)}{(\nu u_0)^{1/3}},\ Gc = \frac{g\beta(C' - C'_\infty)}{(\nu u_0)^{1/3}},$$

$$\Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D} \quad (5)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0,\quad \theta = 0,\ C = 0 \text{ for all } Y, t \le 0$$

$$t > 0:\ U = t^2,\quad \theta = t,\quad C = 1 \text{ at } Y = 0 \quad (9)$$

$$U \to 0,\quad \theta \to 0,\quad C \to 0 \quad \text{as} \quad Y \to \infty$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = t\left[(1 + 2\,\eta^2 \Pr)\ erfc(\eta\sqrt{\Pr}) - 2b\sqrt{\Pr}\exp(-\eta^2 \Pr)\right]$$

(10)

$$C = erfc(\eta\sqrt{Sc}) \quad (11)$$

$$U = a\left[(3 + 12\eta^2 + 4\eta^4) erfc(\eta) - b(10 + 4\eta^2)\exp(-\eta^2)\right]$$

$$+ac\left[(3 + 12\eta^2 + 4\eta^4)\, erfc(\eta) - b(10 + 4\eta^2)\exp(-\eta^2)\right.$$

$$\left.-(3+12\eta^2\Pr+4\eta^4(\Pr)^2)\,\mathrm{erfc}(\eta\sqrt{\Pr}) + b\sqrt{\Pr}(10+4\eta^2\Pr)\exp(-\eta^2\Pr)\right]$$

$$+d\,t\left[(1 + 2\,\eta^2)\ erfc(\eta) - 2b\exp(-\eta^2)\right]$$

$$-d\,t\left[(1 + 2\,\eta^2 Sc)\ erfc(\eta\sqrt{Sc}) - 2b\sqrt{Sc}\exp(-\eta^2 Sc)\right]$$

(12)

Where $\eta = \frac{Y}{2\sqrt{t}};\ a = \frac{t^2}{6};\ b = \frac{\eta}{\sqrt{\pi}};\ c = \frac{Gr}{\Pr - 1};\ d = \frac{Gc}{Sc - 1}$

RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters *Gr, Gc, Sc* and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number *Pr* is chosen such that they represent air (*Pr* = 0.71). The numerical values of the velocity are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different values of the Schmidt number (Sc=0.16, 0.3, 0.6, 2.01), Gr = *Gc* = 5 and *t* = 0.4 are studied and presented in figure 1. It is observed that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

Figure 2 demonstrates the effects of different thermal Grashof number (*Gr* = 2, 5) and mass Grashof number (*Gc* = 5,10) on the velocity at *t* = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The velocity profiles for different values of (t = 0.2, 0.4, 0.6,0.8) and *Gr* = *Gc* = 5 are shown in figure 3. In this case, the velocity increases gradually with respect to time t.

Figure 4 represents the value of the temperature gradually decreases time (t=0.2 ,0.4 , 0.6, 0.8) and Sc=0.6. The trend of the temperature reaches the parameter η after a certain period of time.

Figure 5 illustrate the concentration profile for different Schmidth number can be illustrated. The effect of concentration here decreases in a monotone style from their respective values to a zero value. The concentration increases with decreasing value of Schmidth number (Sc = 0.16 , 0.3 , 0.6 , 2.01)

NOMENCLATURE

A Constants
C' species concentration in the fluid kgm^{-3}
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions

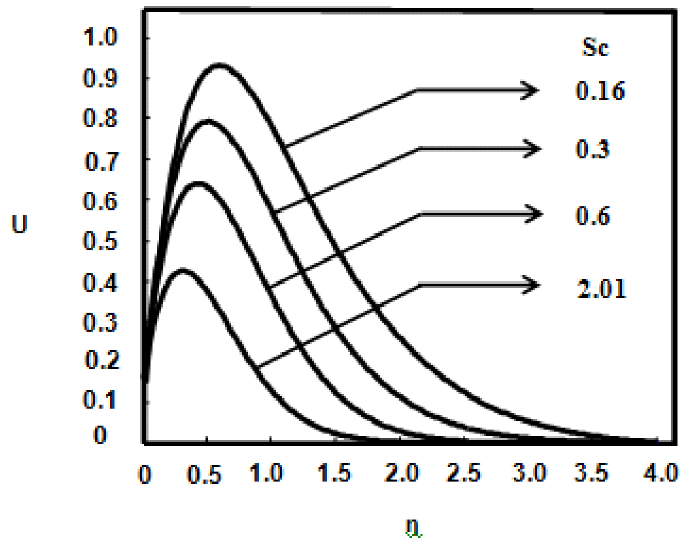

Figure 1 : Velocity profile for different values of Sc

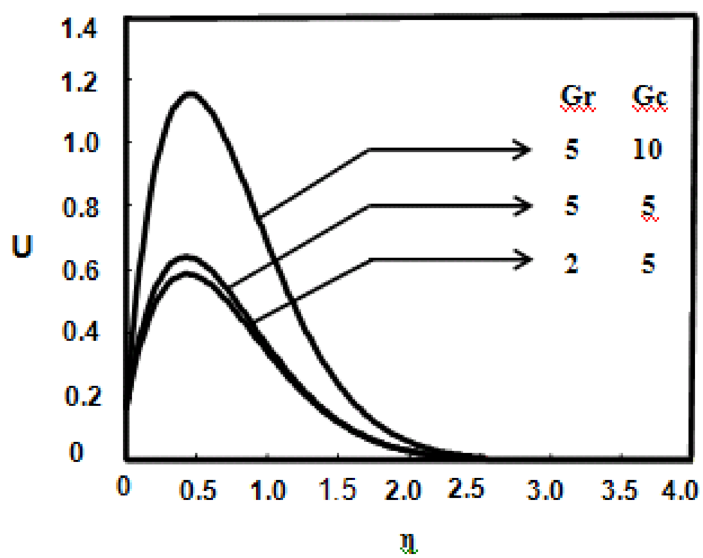

Figure 2 : Velocity Profile for different values of Gr, Gc

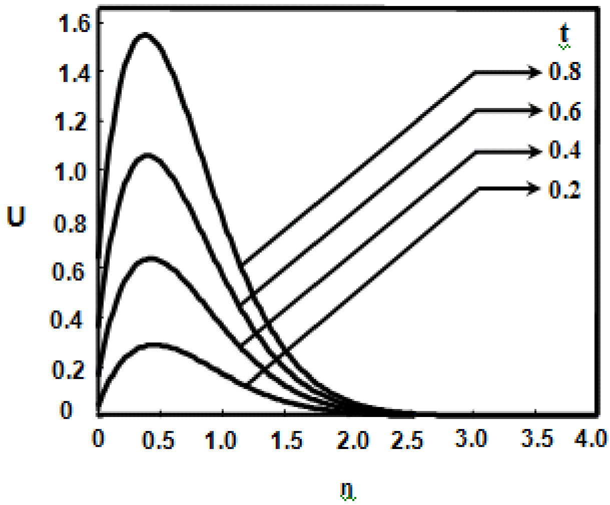

Figure 3 : Velocity profile for different values of t

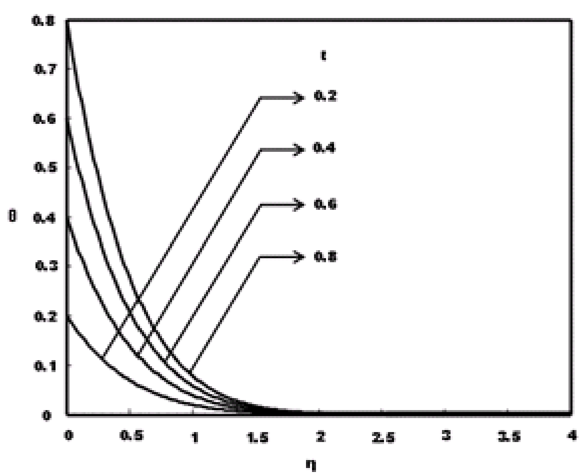

Figure 4 : Temperature Profile for different t

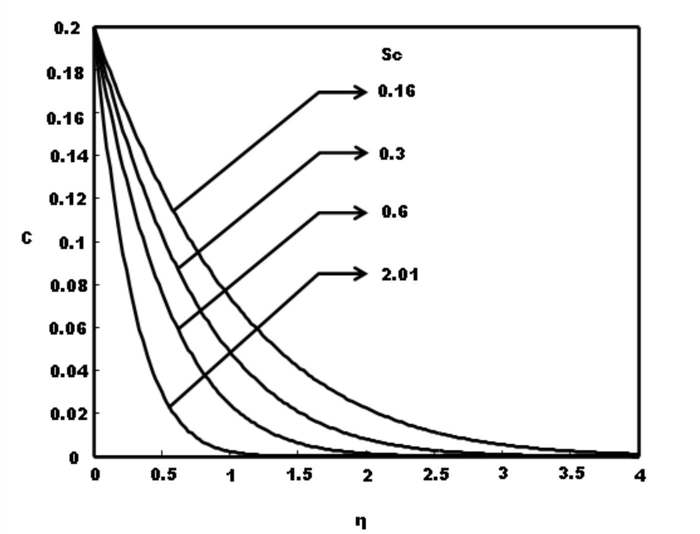

Figure 5 : Concentration Profile for different Sc

CONCLUSION

The parameters are thermal Grashof number, mass Grashof number, Schmidt number and time studied graphically.

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number.

(ii) The velocity increases with increasing values of time t, but the trend is just reversed with respect to the Schmidt number.

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