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Magnetic Field - A Catalyst for Instability

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Abstract : The paper critically examined the hydromagnetic stability of dusty swirting flows between two co-axial circular cylinders subject to infinitesimal purturbance of arbitrary orientation. A number of theorems have been established the fact that magnetic field has a dual character on stability of a system.

Keywords : Hydromagnetic stability, Dust particles, Swirling flows

INTRODUCTION

A number of research workers have contributed to the stability of swirling flows between two co-axial cylinders in past few decades. Howard and Gupta **[1]**, Leibovich **[2]**, Chandra and Rathy **[3]**, Lalas **[4]**, Mahesh **[5]** and others have obtained the sufficient conditions of stability in the form of Richardson criterion. Leibovich defined the effective Richardson number so as to include the effect of density variations. Chandra and Rathy defined the effective Richardson number to take care of perturbations of arbitrary orientation and Lalas defined the effective Richardson number consistent with the stability of swirling flows to the infinitesimal adiabatic perturbations of arbitrary orientation.

Sharma **[6]** has studied the instability of the plane interface between two Oldroydian visco-elastic superposed conducting fluids in the presence of a uniform magnetic field.

Sharma and Kumar **[7]** have made an attempt to study the hydromagnetic Rayleigh-Taylor instability of Oldroydian visco-elastic fluid in a porous medium in the presence of a variable magnetic field. Recently, Goel and Agrawal [8J have made numerical investigations of the hydromagnetic thermal convection in a visco-elastic dusty fluid in a porous medium and they have shown that the principle of Exchange of Stabilities is valid at the marginal state under certain conditions.

Generally, the magnetic field has a stabilizing character, however there are a few exceptions. For example, Kent **[9]**, Gilman **[10]** and Jain and Agrawal **[11]** have obtained unstable wave number ranges in the presence of a magnetic field which were known to be stable in its absence, showing thereby, that magnetic field acts as catalyst for instability in certain situations. This dual character of magnetic field has made the hydromagnetic stability of flows much more meaningful and interesting.

Lesson, Fox and Zien **[12]** have concentrated their study upon subsonic and supersonic disturbances. The disturbances are termed as sub-sonic if the sonic velocity C_0 is greater than their perturbation velocity |U – C| and are termed as supersonic if the sonic velocity C_0 is less than their relative perturbation velocity.

The expression

$$A = V_A^2 / \Omega^2,$$

where $V_A^2 = H^2 / \mu_0\rho_0$ is the square of Alfven velocity.

and $\Omega^2 = (W-(mV)|(\gamma K)-C)^2$ is the square of the relative velocity of the perturbations, occur in our study. This expression is similar to the one found in Lesson, Fox and Zien and in the similar analysis of above authors.

In what follows, we assume that the Alfven velocity is much less than the relative velocity of perturbations so that $V_A << | W - \frac{mv}{rK} - C |$. The validity of this assumption is justified and can not be questioned for the small applied magnetic field. Then, in the expression $1 - | V_A^2 / \Omega^2 |, | V_A^2 / \Omega^2 |$ can be neglected as compared to 1. Equations are considerably simplified still retaining the effect of small magnetic field.

In this chapter, we have examined the hydromagnetic stability of dusty swirling flows between two co-axial circular cylinders subject to infinitesimal perturbations of arbitrary orientation. It has been established by Saffman **[13]** and others that the fine dust destabilizes and the coarse dust stabilizes the flows and a situation predicted to be stable in the presence of very fine dust particles is expected to remain stable even in the presence of coarse dust particles. We have, therefore, investigated the reaction of the system to infinitesimal perturbations of arbitrary orientation only in the presence of very fine dust particles. Following Saffman, we have assumed the dust particles to be spherical in shape and uniform in size. The concentration of dust particles varies in the radial direction, so the number density N depends upon r. The sedimentation effects are neglected throughout the present analysis.

EQUATIONS OF MOTION

The equations governing the motion of an incompressible, non-dissipative hydromagnetic dusty swirling flows in cylindrical polar coordinates are :

For clean fluid:

$$\rho\left[\frac{\partial u_r}{\partial t} + (\mathbf{u}.\nabla)u_r - \frac{u_\theta^2}{r}\right] = \frac{1}{\mu_0}\left[(\mathbf{H}.\nabla)H_r - \frac{H_\theta^2}{r}\right] - \frac{\partial \Pi}{\partial r} + K*N(v_r - u_r), \quad (1)$$

$$\rho\left[\frac{\partial u_\theta}{\partial t} + (\mathbf{u}.\nabla)u_\theta - \frac{u_r u_\theta}{r}\right] = \frac{1}{\mu_0}\left[(\mathbf{H}.\nabla)H_\theta - \frac{H_\theta H_r}{r}\right] - \frac{1}{r}\frac{\partial \Pi}{\partial \theta} + K*N(v_\theta - u_\theta), \quad (2)$$

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$$\rho\left[\frac{\partial u_z}{\partial t}+(\mathbf{u}.\nabla)u_z\right]=\frac{1}{\mu_0}[(\mathbf{H}.\nabla)H_z]-\frac{\partial \Pi}{\partial z}+K*N(v_z-u_z) \quad (3)$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0, \quad (4)$$

$$\frac{\partial \rho}{\partial t}+u_r\frac{u\rho}{\partial r}+\frac{u_\theta}{r}\frac{\partial \rho}{\partial \theta}+u_z\frac{\partial \rho}{\partial z}=0 \quad (5)$$

and
$$\frac{\partial H_r}{\partial t}+\frac{H_r}{r}+\frac{1}{r}\frac{\partial H_\theta}{\partial \theta}+\frac{\partial H_z}{\partial z}=0 \quad (6)$$

For Dust Particles

$$mN\left[\frac{\partial v_r}{\partial t}+(\mathbf{v}.\nabla)v_r-\frac{v_\theta^2}{r}\right]=-mNg+K*N(u_r-v_r)+\frac{1}{\mu_0}\left[(\mathbf{H}.\nabla)H_r-\frac{H_\theta^2}{r}\right], \quad (7)$$

$$mN\left[\frac{\partial v_\theta}{\partial t}+(\mathbf{v}.\nabla)v_\theta+\frac{v_r v_\theta}{r}\right]=K*N(u_\theta-v_\theta)\frac{1}{\mu_0}\left[(\mathbf{H}.\nabla)H_\theta-\frac{H_\theta H_r}{r}\right], \quad (8)$$

$$mN\left[\frac{\partial v_z}{\partial t}+(\mathbf{v}.\nabla)v_z\right]=K*N(u_z-v_z)+\frac{1}{\mu_0}[(\mathbf{H}.\nabla)H_z] \quad (9)$$

$$\frac{\partial v_r}{\partial t}+\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0 \quad (10)$$

and
$$\frac{\partial N}{\partial t}+v_r\frac{\partial N}{\partial r}+\frac{v_\theta}{n}\frac{\partial N}{\partial \theta}+v_z\frac{\partial H}{\partial z}=0 \quad (11)$$

where t is the time, □ is the pressure, k = 6□ □a is the Stoke's resistance coefficient, a being the radius of dust particles assumed to be spherical; M and N(r) are respectively the mass and the number density of dust particles. It is assumed that the gravitational force acts in the radial direction only on the dust particles, u and v are respectively the velocities of clean fluid and dust particles. H_r, H□ and H_z are the components of magnetic field in the radial, azimuthal and axial directions. Equations (1) to (4) are satisfied in the cylindrical region $R_1 \le r \le R_2$ by the stationary state (time independent) solution defined as

$$\left.\begin{aligned}\mathbf{u}&=V(r)e_\theta+W(r)e_z\\ \mathbf{v}&=V(r)e_\theta+W(r)e_z\\ \mathbf{H}&=(0,0,H)\end{aligned}\right] \quad (12)$$

subject io the requirements

$$\frac{\rho v^2}{r}=\frac{\partial \Pi}{\partial r} \quad \text{and} \quad v^2=gr.$$

LINEARIZED PERTURBATION EQUATIONS

Let the basic state given by (12) be slightly perturbed, so that in the perturbed state, the velocity, density and pressure of clean gas and the dust particles are given by

$$\begin{aligned}\mathbf{u}'&=[u_r', v(r)+u'(\theta), W(r)+u'(z)],\\ \mathbf{v}'&=[v_r', V(r)+V'(\theta), W(r)+v'(z)],\\ \mathbf{H}&=(h_r', h'(\theta), H+h_z'),\\ \rho&=\rho_0+\rho',\\ N&=N_0(r)+N'\end{aligned}$$

and $\Pi'=\Pi_0+\delta\Pi.$

Following the usual procedure and normal mode technique given by

$$f'(r,\theta,z,t)=f(r)e^{[i(pt-m\theta-kz)]},$$

where $p = p_r + ip_i$ is the complex frequency and m and k are respectively the azimuthal and axial wave numbers, both taken to be real. Taking this dependence of perturbations on r, □ , z and t.

Using the approximation,

$$v_r=u_r-\frac{N}{N_0}g\tau,$$

$$v_\theta=u_\theta$$

and $v_z=u_z.$

we get

$$\rho_0\left[\Omega\left(\frac{im}{r}u_\theta+iku_z\right)+\left(DV+\frac{V}{r}u_r+k(DW)\right)u_r\right]=-\frac{HK}{\mu_0}\left(i\frac{m}{r}h_\theta+kh_z\right)+\frac{i\Pi'}{S} \quad (13)$$

where $S=r^2/(m^2+k^2r^2)$

On using equation (4), equation (13) becomes

$$S\rho_0\left[\Omega\left(Du_r+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]-\frac{SH^2K^2}{\mu_0}\left(D\left(\frac{\mu_r}{\Omega}\right)+\frac{u_r}{r\Omega}\right)=i\Pi' \quad (14)$$

$\Rightarrow$

$$u_\theta=\frac{\frac{m}{r}\left\{S\rho_0\left[\Omega\left(Du_r+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]-\frac{SH^2k^2}{\mu_0}\left(D\left(\frac{u_r}{r}\right)+\frac{u_r}{r\Omega}\right)\right\}-\rho_0\left(DV+\frac{V}{r}\right)u_r+\frac{H^2k^2}{\mu_0\Omega^2}\left(DV-\frac{V}{r}\right)u_r}{i\rho_0\left(1-\frac{\Omega_A^2}{\Omega^2}\right)}$$

□ □ □ □

where $\Omega_A^2=\frac{H^2k^2}{\rho_0\mu_0}.$

Hence,

$$u_\theta=\frac{\left\{i\Omega u_r-\frac{2V}{r}\left[\frac{m}{r}\left[S\rho_0\Omega\left(Du_r+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]\right]-\frac{SH^2k^2}{\mu_0}\left(D\left(\frac{u_r}{r}\right)+\frac{u_r}{r\Omega}\right)\right\}-\rho_0\left(DV+\frac{V}{r}\right)\left(1-\frac{\Omega_A^2}{\Omega^2}\right)u_r-\frac{2H^2k^2vu_r}{\mu_0\Omega^2 r}}{i\rho_0\left(1-\frac{\Omega_A^2}{\Omega^2}\right)}$$

$$=\frac{\rho v^2}{r}+iD\left\{\left[S\rho_0\Omega\left(Du_r+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]-\frac{SH^2k^2}{\mu_0}\left(D\left(\frac{u_r}{r}\right)+\frac{u_r}{r\Omega}\right)\right\}$$

$$-kN'g\tau+\frac{iH^2k^2u_r}{\mu_0\Omega}.$$

Also, we have

$$i\Omega\rho'+u_r D\rho_0=0 \quad \text{or}$$

$$\rho'=-\left[\frac{u_r(D\rho_0)}{i\Omega}\right].$$

On eliminating □ ′ from equation (15) we get

$$\left[D-\frac{2mv}{r^2\Omega[1-(\Omega_A^2/\Omega^2)]}\right]\left\{\left[S\rho_0\Omega\left(Du_r+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]-\frac{SH^2k^2}{\mu_0}\left(D\left(\frac{u_r}{\Omega}\right)+\frac{u_r}{r\Omega}\right)\right\}$$

$$-\rho_0\Omega u_r+\frac{2\rho_0 v}{r\Omega}\left(DV+\frac{V}{r}\right)u_r+\frac{4H^2k^2V^2}{\mu_0\Omega^2r^2}\frac{u_r}{\Omega[1-(\Omega_A^2/\Omega^2)]}+\frac{v^2(D\rho_0)}{r\Omega}u_r$$

$$+\frac{H^2k^2}{\mu_0\Omega}u_r+ikN'g\tau=0$$

or

$$\left[D-\frac{2mv}{r^2\Omega[1-(\Omega_A^2/\Omega^2)]}\right]\left\{\left[S\rho_0\Omega\left(\frac{du_r}{dr}+\frac{u_r}{r}\right)+\left(DV+\frac{V}{r}\right)\frac{m}{r}u_r+k(DW)u_r\right]-\frac{SH^2k^2}{\mu_0}\left(\frac{d}{dr}\left(\frac{u_r}{\Omega}\right)+\frac{u_r}{r\Omega}\right)\right\}$$

$$-\rho_0\Omega u_r+\frac{2\rho_0 v}{r\Omega}\left(DV+\frac{V}{r}\right)u_r+\frac{4H^2k^2V^2}{\mu_0\Omega^2r^2}\frac{u_r}{\Omega[1-(\Omega_A^2/\Omega^2)]}+\frac{N^2\rho_0u_r}{\Omega}$$

$$+\frac{H^2k^2}{\mu_0\Omega}u_r-\frac{k*DN_0u_rg\tau}{\left(\Omega+\frac{iDN_0}{N_0}g\tau\right)}=0$$

where $N^2=\frac{D\rho_0}{\rho_0}\frac{v^2}{r}.$

Further, equation (11) yields

$$N'=\frac{-v_r(DN_0)}{i\Omega}$$

or

$$i\Omega N'=-\left(u_r-\frac{N}{N_0}g\tau\right)(DN_0)$$

or

$$N'=\frac{i(DN_0)u_r}{\left(\Omega+\frac{iDN_0}{N_0}g\tau\right)}.$$

Finally, equations (18) and (19) yield after some simplifications

$$\left[D-\frac{2mv}{r^2\Omega[1-(\Omega_A^2/\Omega^2)]}\right]\left\{\left[S\rho_0\Omega\left(\frac{du_r}{dr}+\frac{u_r}{r}\right)+\left(V'+\frac{V}{r}\right)\frac{m}{r}\frac{u_r}{\Omega}+\frac{kW'u_r}{\Omega}\right]-\frac{SH^2k^2}{\mu_0}\left(\frac{d}{dr}\left(\frac{u_r}{r}\right)+\frac{u_r}{r\Omega}\right)\right\}$$

$$-\rho_0\Omega\left[1-\frac{\Omega_A^2}{\Omega^2}-\frac{N^2}{\Omega^2}-\frac{\Phi}{\Omega^2}-\frac{iQ}{\rho_0\Omega}\right]u_r+\frac{4H^2k^2V^2u_r}{\mu_0\Omega^3r^2(1-(\Omega_A^2/\Omega^2))}=0$$

where $Q=\frac{ik*(DN_0)g\tau}{\left(\Omega+\frac{iDN_0}{N_0}g\tau\right)}$ and $\Phi=\frac{2V}{r}\left(DV+\frac{V}{r}\right)$ is the Rayleigh discriminant.

The boundary conditions are $u_r = 0$ at $r = R_1$ and R_2.

Using the transformation

$$u_r = □^{1/2}□ .$$

In equation (20), we get

$$D\left[S\rho_0\Omega\left\{\left(1-\frac{\Omega_A^2}{\Omega^2}\right)\left(\psi'+\frac{\psi}{r}\right)\right\}\right]+D\left[S\rho_0\left\{\frac{2mv}{r^2}-\frac{\Omega'}{2}\left(1-\frac{\Omega_A^2}{\Omega^2}\right)\right\}\right]\psi$$

$$-\frac{S\rho_0\Omega'^2}{4\Omega}\left(1-\frac{\Omega_A^2}{\Omega^2}\right)\psi+\frac{\Omega'}{2}S\rho_0\left[\left(1-\frac{\Omega_A^2}{\Omega^2}\right)+\frac{4mrv}{\Omega r}\right]\frac{\psi}{r}-\frac{S\rho_0 2mV}{r^2}$$

$$\left[1+\frac{2mV}{r\Omega[1-(\Omega_A^2/\Omega^2)]}\right]\frac{\psi}{r}+\frac{4V^2\Omega_A^2\psi}{r^2\Omega^3[1-(\Omega_A^2/\Omega^2)]}$$

$$-\rho_0\Omega\left[1-\frac{\Omega_A^2}{\Omega^2}\frac{N^{*2}}{\Omega^2}-\frac{\Phi}{\Omega^2}-\frac{iQ}{\rho_0\Omega}\right]\psi=0.$$

Multiply equation (21) by r□ *, where □ * is the complex conjugate of □ * and integrate over the range of r, we have

$$\int\left[-S\rho_0\Omega\left(1-\frac{\Omega_A^2}{\Omega^2}\right)r\left|\psi'+\frac{\psi}{r}\right|^2\right]-\frac{\Omega' S\rho_0}{4\Omega}\left(1-\frac{\Omega_A^2}{\Omega^2}\right)r+|\psi|^2+\frac{\Omega'}{2}S\rho_0\left[\left(1-\frac{\Omega_A^2}{\Omega^2}\right)+\frac{4mv}{r\Omega}\right]|\psi|^2$$

$$+\frac{S\rho_0}{2}\left[\Omega'\left(1-\frac{\Omega_A^2}{\Omega^2}\right)-\frac{4mV}{r^2}\right][|\psi|^2+(\psi\psi^*)'r]-\frac{2S\rho_0 mV}{r^2}\left[1+\frac{2mv}{r\Omega[1-(\Omega_A^2/\Omega^2)]}\right]|\psi|^2$$

$$+\frac{4V^2\Omega_A^2}{r^2\Omega^3[1-(\Omega_A^2/\Omega^2)]}|\psi|^2-\rho_0\Omega r\left[\left(1-\frac{\Omega_A^2}{\Omega^2}-\frac{N^2}{\Omega^2}-\frac{\Phi}{\Omega^2}-\frac{iQ}{\rho_0\Omega}\right)|\psi|^2\right]dr=0.$$

(19)

Taking $1>>\frac{\Omega_A^2}{\Omega^2}$, we have

$$\int\left[-S\rho_0\Omega r\left|\psi'+\frac{\psi}{r}\right|^2\right]-\frac{\Omega'^2 S\rho_0 r|\psi|^2}{4\Omega}+\frac{\Omega'}{2}S\rho_0\left[1+\frac{4mv}{r\Omega}\right]|\psi|^2$$

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$$+\frac{S\rho_0}{4}\left[\Omega'\left(1-\frac{\Omega^2}{\Omega_A^2}\right)-\frac{4mV}{r^2}\right][|\psi|^2+(\psi\psi^*)'r]-\frac{2S\rho_0 mV}{r^2}\left[1+\frac{2mv}{r\Omega[1-(\Omega_A^2/\Omega^2)]}\right]|\psi|^2$$

□ □

Substitution of the expressions

$$+\frac{4V^2\Omega_A^2}{r^2\Omega^3[1-(\Omega_A^2/\Omega^2)]}|\psi|^2-\rho_0\Omega r\left[\left(1-\frac{\Omega_A^2}{\Omega^2}-\frac{N^2}{\Omega^2}-\frac{\Phi}{\Omega^2}-\frac{iQ}{\rho_0\Omega}\right)|\psi|^2\right]dr=0.$$

$$\Omega=p-\frac{mV}{r}-kW,\ \Omega'=-\frac{mV'}{r}+\frac{mV}{r^2}-kW' \text{ and } Q=\frac{i(DN_0)k^*g\tau}{\left(\Omega+i\frac{DN_0}{N_0}g\tau\right)}$$

into equation (23), we get

$$\int\left\{\left[-S\rho_0\left(p_r-\frac{mV}{r}-kW+ip_i\right)\left(r\left|\psi'+\frac{\psi}{2}\right|^2\right)-\frac{\Omega'^2}{4|\Omega|^2}\left(p_r-\frac{mV}{r}-kW-ip_i\right)S\rho_0 r|\psi|^2\right]\right.$$

$$+\frac{\Omega' S\rho_0}{2}\left[2|\psi|^2+r\psi\psi^*+\frac{4mV}{r|\Omega|^2}\left(p_r-\frac{mV'}{r}-kW-ip_i\right)|\psi|^2\right]$$

□ □

$$-2S\rho_0 mV\left[2|\psi|^2+r\psi\psi^*+\frac{2mv}{r|\Omega|^2}\left(p_r-\frac{mV}{r}-kW-ip_i\right)|\psi|^2\right]$$

$$-\rho_0 r\left[\left(p_r-\frac{mV'}{r}-kW+ip_i\right)-\frac{N^2}{|\Omega|^2}\left(p_r-\frac{mV}{r}-kW-ip_i\right)-\frac{\Phi}{|\Omega|^2}\left(p_r-\frac{mV}{r}-kW-ip_i\right)\right]$$

$$+DN_0k^*g\tau\frac{\left[p_r-\frac{mV}{r}-kW-i\left(p_i+\frac{DN_0}{N_0}g\tau\right)\right]|\psi|^2}{\rho\left|\left(p_r-\frac{mV}{r}-kW\right)+i\left(p_i+\frac{DN_0}{N_0}g\tau\right)\right|}$$

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$$+\frac{4V^2\Omega^2}{|\Omega|^2}\left[\left(p_r-\frac{mV}{r}-kW\right)-3\left(p_r-\frac{mV}{r}-kW\right)P_i^3\right]$$

$$\left.+i\left[p_i^3-3\left(p_r-\frac{mV}{r}-kW\right)p_i|\psi|^2\right]\right\}dr=0$$

RESULTS AND DISCUSSIONS

Theorem 1: Neutral modes do not exist.

Proof: The imaginary part of equation (24) is given by

$$p_i(I_1+I_2+I_3+I_4)=I_5+I_6p_i^3$$

where $I_1=\int\rho_0 Sr\left|\psi'+\frac{\psi}{r}\right|^2dr,$

$$I_2=\int r\rho_0|\psi|^2dr,$$

$$I_3=\int\frac{\rho_0 r}{|\Omega|^2}\left\{N^2+\Phi-\frac{DN_0k^*g\tau(\Omega)^2}{\rho_0\left|p_r-\frac{mV}{r}-kW+i\left(p_i+\frac{DN_0}{N_0}g\tau\right)\right|^2}-S\left(\frac{\Omega'}{2}-\frac{2mV}{r^2}\right)\right\}|\psi|^2dr,$$

$$I_4=\int 2\left(p_r-\frac{mV}{r}-kW\right)^2\frac{V^2\Omega_A^2}{|\Omega|^6}|\psi|^2dr,$$

$$I_5=\int\frac{\rho_0 r(DN_0)^2k^*g\tau|\psi|^2}{N_0\left|\left(p_r-\frac{mV}{r}-kW\right)+i\left(p_i+\frac{DN_0}{N_0}g\tau\right)\right|^2}dr,$$

and $I_6=\int\frac{4V^2\Omega_A^2|\psi|^2}{|\Omega|^6}dr$

Since I_5 is a positive definite integral, therefore p_i cannot be zero in view of equation (25). It follows that the neutral modes (p_i = 0) do not exist in the system. Therefore, the modes are either stable or unstable.

In the following theorem, we prove that the modes are in fact unstable.

Theorem 2 : Modes are unstable.

Proof : Equation (25) is a cubic equation in p_i and if p_{i1}, p_{i2} and p_{i3} are the roots of this equation, then

$$p_{i1}p_{i2}p_{i3}=-I_5/I_6.$$

Since the integrals I_5 and I_6 are both positive definite, therefore the product of the roots is negative implying, thereby, that either one root or all three roots are negative. But since the sum of the roots is zero, therefore all these roots can not be negative. Hence one root is •negative which ensures the existence of one unstable mode.

Hence, an unstable situation is predicted in the presence of a magnetic field, which was stable in its absence.

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