

Structural Analysis of Three Stage Coupled Planetary Gear Train and Determination of Efficiency

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Abstract: Planetary gear transmissions are compact, high power transmitting speed reductions technology. Structural analysis of three stage coupled planetary gear train is reviewed. This analysis depicts the fast and easy determination of the speed ratio, torques, and efficiency and power flow directions of coupled planetary gear trains. Three stage efficiency determined.

Keywords: Efficiency, planetary gear trains, structural analysis, lever analogy, torque, ratio.

1. Introduction:

The planetary gear trains are widely used in many industrial, automotive, aerospace and marine applications. The Planetary gear has wide industrial acceptability, because of compact power density. Planetary gear train has various ratios possibility by changing different coupling options.

The range of possible reduction ratios is depends on the relative size of the planets. For single plane transmission, the planet gear has no size at a ratio of two. As the ratio increases, size of the planet increases relative to the sizes of sun and ring. Planetary gear ratio largely depends on number of planet used in each stage. The Number of planet increases torque transmitting capacity and service life increases but maximum ratio availability decreases.

Traditional methods of analyzing planetary gear trains, by means of torque and speed calculations, are complex and take time. A simpler method of analyzing and characterizing planetary gear trains is called the Lever Analogy Diagrams which are commonly utilized in industry.

H. Benford and M. Leising [1] present a convenient and effective concept to help simplify transmission calculations. With this tool, the entire transmission is usually represented by a single lever, and the calculation of most torque, speed and reaction characteristics is as simple as summing the moments of a lever.

Raghavan M. [5] proposed a novel analysis tool for planetary gear train representation. This extends the traditional concept of a lever representation of a planetary gear set to one that includes negative lever ratios. The analytical work reveals exhaustive permutation of the node of lever leading to all possible topological arrangements of the planetary gear train.

The development of an engineering method for structural analysis of multi-carrier planetary gear trains is presented by Karaivanov D. [11]. This method is based on lever analogy between the simple (one-carrier) and coupled (multi-carrier) planetary gear trains. The method is simple to determine the ratios, as well as for establishing the existence of inner division or outer circulation of power, which is an important precondition

for the true determination of the efficiency of these gears. The method allows the fast and easy determination of the speed ratio, torques and efficiency and power flow directions of coupled planetary gear trains.

Giger U., and Arnaudov K. [12] present patented new design wind turbine gearbox. In order to develop its optimum design features new way of gearbox designs are proposed with easy determination of the speed ratio, torques and efficiency and power flow directions of coupled planetary gear trains based lever analogy.

S. Troha, D. Karaivanov [8] reviews experiments performed with a coupled two-carrier planetary gear train with four external shafts and two brakes. The losses in the gear train are determined by means of static loading. A check is made on the validity of the relations deduced for determination of the gear train's efficiency as a function of the efficiencies of its coupling gear trains.

Structural analysis of compound planetary gear trains is much facilitated if the coupling simple planetary gears are represented with the structural symbol of Wolf [9] (a circle with three shafts) in which Arnaudov depicts the three external shafts with lines of different thickness according to the magnitudes of their corresponding torques (Fig. 1).

In this paper the structural analysis of three stages coupled planetary gear train is reviewed. This analysis is based on lever analogy that allows the fast and easy determination of the speed ratio, torques and power flow directions of coupled planetary gear trains. The efficiency in each stage is determined based on parameters which dominate planetary gear ratio. Three stage planetary gear trains of various coupling permutation for different applications are studied. The power losses of sun-planet and planet-ring mesh of three stage planetary gear train is determined.

2. Lever Analogy:

Levers – The basic building block of the analogy is the lever which replaces the planetary gear set. The lever proportions are determined by the numbers of teeth on (or the working radii of) the sun gear and annulus gear.

The lever analogy diagram is very useful in analyzing gear train that has more than two connected planetary gear sets. For a single planetary gear set, it is no need to add a level of abstraction. The lever analogy is a translational-system representation of the rotating parts for the planetary gear. In the lever analogy, an entire compound planetary gear train can usually be represented by a single horizontal lever. The input, output and reaction torques are represented by vertical forces on the lever. The lever motion, relative to the reaction point, represents rotational velocities.

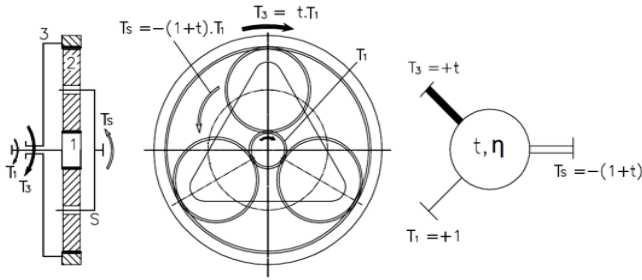


Fig 1: Elementary one carrier Planetary Gear.

The torques of the three external shafts are depicted in Fig.1. They are in a strictly defined ratio, irrespective of operating mode of the gear trains

- 1) with 1 degree of freedom (as a reducer or multiplier)
- 2) with 2 degrees of freedom (differentiating or summing).

When the losses are disregarded, i.e. efficiency $\eta = \eta_{13} = \eta_{31} = 1$, this ratio is as follows:

$$T_1 : T_3 : T_S = T_1 : t.T_1 : -(1+t).T_1 = +1 : +t : -(1+t) \quad (1)$$

Where:

T_1 = Torque of sun gear's shaft

T_3 = Torque of ring gear's shaft

T_S = Torque of carrier shaft

$$t = \text{Torque ratio of gear train } t = \frac{T_3}{T_1} = \left| \frac{Z_3}{Z_1} \right|$$

Z_1 = Sun gear number of teeth

Z_3 = Ring gear number of teeth

The three stages planetary gear train arrangement with carrier coupled with next stage sun gear by means of spline and fixed ring gears gives maximum transfer ratio (Fig. 3).

Sectional arrangement of three stage planetary gearbox is represented in fig. 2.

Simple one carrier planetary gear has three external shafts according to Wolf's symbol (Fig 1).

The three stage coupled planetary gear train is with 1 degree of freedom with two free shaft 1 and SIII (Fig 3).

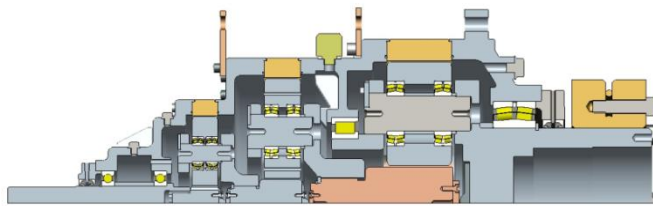


Fig. 2: Sectional arrangement of three stage coupled planetary gearbox.

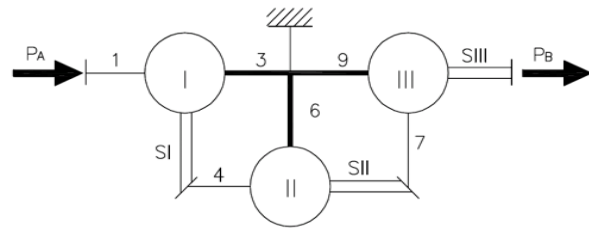
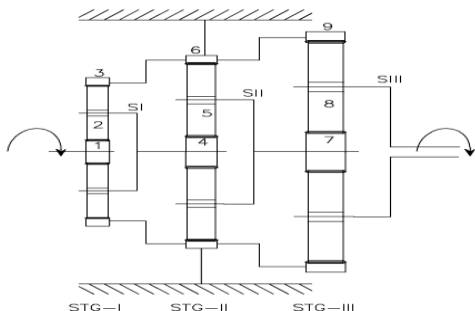


Fig 3: Structural Wolf's diagram of the three stage coupled planetary gear train.

3. Determination of Speed Ratio:

The torques are denoted on the corresponding shafts, beginning with one of the sun gears, which is assigned a torque equal to +1. The torques of the other shafts is determined through the torque ratio t by the formulae (1).

Torques ratios of three stages are t_1, t_2 & t_3 respectively. Torques of each member of three stage coupled planetary gear train are calculated using individual torques ratios t_1, t_2 & t_3 . The order of the torque calculation is marked by the circled numbers in fig (4).

When the losses are not being considered $T_A \cdot \omega_A = T_B \cdot \omega_B$ and so the kinetic speed ratio could be obtained from torque T_A of the input and T_B of the output shafts determined with no regard to losses ($\eta = 1$).

$$i_k = \frac{\omega_A}{\omega_B} = - \frac{T_B}{T_A} \quad (\eta = 1) = f(t_1, t_2, t_3 \dots) \quad (2)$$

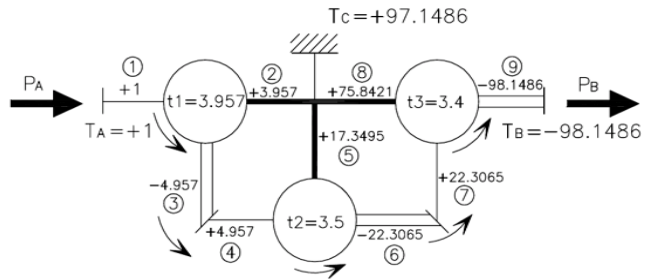


Fig. 4: Verification of the ideal torques of the coupled three carrier planetary gear train.

The transfer speed ratio i_k for the three stages coupled planetary gear train as per fig (3)

$$i_k = - \frac{T_B}{T_A} = - \frac{-98.1486}{+1} = 98.1486 \quad (3)$$

4. Power analysis:

Power losses in the gear train are determined with efficiency

$$\eta = \frac{P_B}{P_A} = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A} < 1 \quad (4)$$

In the present work the direction of the internal power flow from sun gear to ring gear. Assuming the sun gear torque $T_1 = +1$, the ring gear torque is $T_3 = +t$.

The most reliable determination of efficiency of gear drives are doubtlessly the experimental one. But determination of efficiency by experiment for heavy power transmission gear

drives is very difficult. Theoretical determination have important role for efficiency calculation. To determine the stationary efficiency η_1, η_2 & η_3 for the three single-carrier planetary gear trains, the simple formula by Foster [12], which is primarily, take the dominant impact of the number of teeth into account. With respect to the first single-carrier planetary gear train, the formula of efficiency

$$\eta = 1 - \left[0.15 * \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) + 0.2 * \left(\frac{1}{Z_2} - \frac{1}{\|Z_3\|} \right) \right] \quad (5)$$

Where

- Z_1 = Ring gear no. of teeth
- Z_2 = Planet gear no. of teeth
- Z_3 = Sun gear no. of teeth

The torque of the shaft is determined with losses following the order assumed in the kinematic analysis (Fig. 5). The transfer ratio i_T for three stages coupled planetary gear train is

$$i_T = \frac{T'_B}{T'_A} = \frac{-95.1021}{1} = -95.1021 \quad (6)$$

The internal power flow directions are shown by arrow marks in fig. 5. The efficiency of the gear train η is

$$\eta = - \frac{i_T}{i_K} = \frac{T'_B}{T'_A} = f(t_1, t_2, t_3, \eta_1, \eta_2, \eta_3, \dots) \quad (7)$$

The efficiency of three stage coupled planetary gear train according to equation (7) is

$$\eta = - \frac{-95.1021}{98.1486} = 0.9689 = 97 \%$$

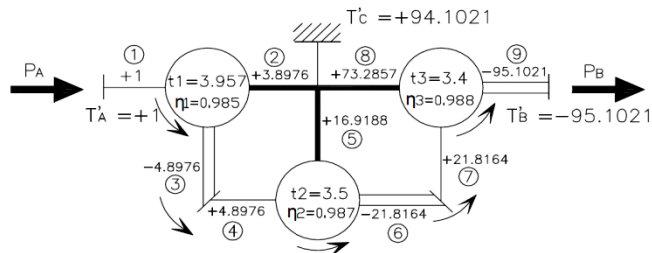


Fig. 5: Verification of the efficiency using real torques with the power flow directions.

5. Conclusions:

The design of this 320 kW three stage planetary gearbox has the following advantages:

- 1) It has compact and maximum power density structural arrangement using multiple planets per train (up to 5).
- 2) Higher safety factors SH (pitting) and SF (bending) of the gears are obtained.
- 3) Selection of geometry parameters to achieve better contact ratio.

The analytical approach described in this paper has the following features.

- 1) Easy calculation of transfer ratio and torque.
- 2) Determination of power flow direction for all coupling combinations.
- 3) Determination of efficiency for coupled three stages planetary gear train.
- 4) Determination of torque of each member of coupled gear train.

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