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Convective Heat Transfer to an Evaporating Liquid Drop in a Direct Contact with an Immiscible Liquid: Theoretical Analysis for Heat Transfer Coefficient and Bubble Growth Rate

Hameed B. Mahood
University of Misan – Misan – Iraq
e-mail: hbmahood@yahoo.com

Abstract: An analytical model for bubble growth and convective heat transfer coefficient of a single liquid/vapour two-phase bubble evaporating in direct a contact with another immiscible liquid media has been developed. The model was based on the solution of the energy equation in a spherical coordinate with a potential flow assumption and a cellular model configuration. A new expression of the convective heat transfer coefficient in terms of the vaporization ratio and the Pe was derived and used to develop the bubble growth rate expression. Different parameters such as the vaporization ratio, Ja, Pe and Fro were tested. The solution results compared well with available experiment data and other theories.

Keywords: Concentric spheres model, Evaporation; Bubble growth, Nusselt number, Direct-contact heat transfer

I. Introduction

Direct contact heat transfer between two immiscible liquids has the advantage of eliminating metallic heat transfer surfaces which are prone to corrosion and fouling. The main features of direct contact heat exchangers are: relative simplicity of design, less scaling problems, higher heat transfer coefficient a bout 20 200 times tan that of single phase flow, higher rates of heat transfer area and capability to operate at relatively low temperature driving forces [1].
Study on evaporating drop in another immiscible liquid appear to have been first studied by Klipstein [2] who obtained experimentally the heat transfer coefficients with an evaporating drop. Sideman and Taitel [3] assumed a potential flow around the evaporating drop and derived a relationship showing the effect of the opening angle on the rate of heat transfer. Prakash and Pinder [4,5] and Adams and Pinder [6] studied experimentally single drop evaporation for different systems and developed empirical relations for the Nusselt number. Tochitani, et al. [7,8] studied experimentally and theoretically the evaporation process of a pentane or furan drop in an aqueous glycerol of high viscosity to maintain the pentane drop close to a spherical shape. Raina and Grover [9,10], Raina et al. [11] and Raina and Wanchoo [12] predicted the total heat transfer rate expression similar to that given by [3,7 8] with modification to the boundary conditions. Voung and Sadhal [13] revealed a great deal of the physics of the problem by numerically analyzing the time dependent Stokes flow field around the evaporating drop in the immiscible liquid. Seetharammu and Battya [14] obtained the correlation for the Nusselt number in terms of Peclet number and Jackob number. Wanchoo [15] predicted the Nusselt number in case of collapsing large two-phase bubble condensing in an quiescent immiscible liquid. Kendoush [16] derived a theoretical equation for calculating quasi-steady state convective evaporation of a rising volatile drop in immiscible liquid with bubble nucleation inside the drop. Mahood [17-20] found a numerical solution and analytical models for the problem of direct contact evaporation and condensation of a single volatile drop in an immiscible liquid with a summing a concentric sphere model.
In the present paper, an analytical model for the heat transfer of a vaporizing two-phase bubble is carried out.

II. Theoretical Analysis

using spherical co-ordinate (r,Θ,φ) , the drop is assume to be a sphere in which the vapor phase is growing symmetrically at the center of the liquid drop. The liquid phase is assumed to be at the boiling point corresponding to the vaporization pressure . The drop is assumed to be moving in a potential flow field. The relative motion of the drop with respect to the continuous phase is represented by U.
The differential equation for steady state heat transfer with axial symmetry is given by:

$$V_r\frac{\partial T}{\partial r}+\frac{V_\theta}{R}\frac{\partial T}{\partial \theta}=\alpha\frac{\partial^2 T}{\partial r^2} \qquad (1)$$

The potential velocity in r direction near the spherical surface of (1) the drop-bubble, leads to the approximation [15].

$$V_r=0 \qquad (2)$$

The tangent component of velocity in V_θ has been given by Kendoush [16] for the evaporating drop in immiscible liquid as:

$$V_\theta=\frac{U}{(1-x)}\left[x-\frac{1}{2}\left(\frac{R}{r}\right)^3\right]sin\theta \qquad (3)$$

Where x represent the vaporization ratio.
Utilizing the binomial theorem and retaining the first two terms :

$$\left(\frac{R}{r}\right)^3\cong 1-\frac{3y}{R} \qquad (4)$$

and

$$\frac{y}{R}\ll 1 \qquad (5)$$

Equation(3) becomes:

$$V_\theta=f_1(x)Usin\theta \qquad (6)$$

Where

$$f_1(x) = \frac{(x+0.5)}{(1-x)} \quad (7)$$

Substituting Eq.(5) into Eq.(1) yields:

$$f_1(x)U\sin\theta \frac{\partial T}{\partial \theta} = R\alpha \frac{\partial^2 T}{\partial y^2} \quad (8)$$

or

$$\sin\theta \frac{\partial T}{\partial \theta} = M \frac{\partial^2 T}{\partial y^2} \quad (9)$$

Where:

$$M = \frac{R\alpha}{Uf_1(x)} \quad (10)$$

The boundary conditions are:

$$T = 0\,,\ \ y = \infty\,,\ \pi > \theta \geq 0 \quad (11)$$

$$T = T_o\,,\ y = 0,\ \pi \geq \theta \geq 0 \quad (12)$$

$$T = 0\,,\ y = 0\,,\ \pi \geq \theta \geq 0 \quad (13)$$

Introducing a new variable as follows:

$$\psi = y\sin^2(\theta) \quad (14)$$

$$\frac{\partial \psi^2}{\partial y^2} = \sin^4\theta \quad (15)$$

$$\phi = \int_0^\theta \sin^3\theta d\theta = \frac{1}{3}\cos^3\theta - \cos\theta + \frac{2}{3} \quad (16)$$

$$\frac{\partial \phi}{\partial \theta} = \sin^3\theta \quad (17)$$

Substituting Eq.(19) and Eq.(17) into Eq.(9), yields:

$$\frac{\partial T}{\partial \phi} = M\frac{\partial^2 T}{\partial \psi^2} \quad (18)$$

With the corresponding boundary conditions as follows:

$$T = 0,\ \psi = \infty, \phi \geq 0 \quad (19)$$

$$T = T_o, \psi = 0,\ \phi \geq 0 \quad (20)$$

$$T = 0, \infty \geq \psi \geq 0, \phi \geq 0 \quad (21)$$

Equation(23) which is parabolic partial differential equation analogous to the diffusion equation. The solution of Eq.(18) is given be Carslaw and Jaeger [21] :

$$T = T_o erfc\left(\frac{\psi}{2(M\phi)^{0.5}}\right) \quad (22)$$

Substituting the values of ψ and ϕ into above equation (Eq.(22)) gives the temperature distribution over the surface of the evaporating drop (two-phase- bubble) as follows:

$$T = T_o erfc\left[\frac{y\sin^2\theta}{2\left(\frac{R\alpha}{Uf_1(x)}\left(\frac{1}{3}\cos^3\theta - \cos\theta + \frac{2}{3}\right)\right)^{0.5}}\right] \quad (23)$$

Hence, local heat flux q_θ is given by:

$$q_\theta = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\left(\frac{\partial T}{\partial \psi}\right)\left(\frac{\partial \psi}{\partial y}\right)_{y=0} \quad (24)$$

$$q_\theta = \frac{kT_0}{\sqrt{\pi}}\frac{\sin^2\theta}{(M\phi)^{0.5}} \quad (25)$$

The local heat transfer coefficient can be found using the relation:

$$h_\theta = \frac{q_\theta}{T_0} \quad (26)$$

Substituting Eq.(25) into Eq.(26) yields:

$$h_\theta = \frac{k\sin^2\theta}{\sqrt{\pi}\left(M\left(\frac{1}{3}\cos^3\theta - \cos\theta + \frac{2}{3}\right)\right)^{0.5}} \quad (27)$$

The local Nusselt number can be given as:

$$Nu_\theta = \frac{\sqrt{2}}{\sqrt{\pi}} f(x)Pe^{0.5}\frac{\sin^2\theta}{\left(\frac{1}{3}\cos^3\theta - \cos\theta + \frac{2}{3}\right)^{0.5}} \quad (28)$$

Where:

$$f(x) = \left(\frac{x+0.5}{1-x}\right)^{0.5} \quad (29)$$

The average Nusselt number now can be found using the following relation:

$$Nu = \frac{1}{2}\int_0^\pi Nu_\theta \sin\theta\, d\theta \quad (30)$$

Substituting Eq.(28) into Eq.(30) yields:

$$Nu = \frac{\sqrt{2}}{2\sqrt{\pi}} f(x)\, Pe^{0.5} \int_0^\pi \frac{\sin^3\theta\, d\theta}{\sqrt{\phi}} \quad (31)$$

To solve the integration appearing in Eq.(31) above , let:

$$\phi = \frac{1}{3}\cos^3\theta - \cos\theta + \frac{2}{3} \quad (32)$$

$$d\phi = (-\cos^2\theta \sin\theta + \sin\theta)d\theta = \sin^3\theta\, d\theta$$

(33)

And the integration limits are transformed as follows:

$$\theta = \beta,\ \phi = 0 \quad (34)$$

$$\theta = \pi,\ \phi = \frac{4}{3} \quad (35)$$

Now, Eq.(31) becomes:

$$Nu = \frac{1}{\sqrt{2\pi}} f(x)\, Pe^{0.5} \int_0^{\frac{4}{3}} \frac{d\phi}{\sqrt{\phi}} \quad (36)$$

Complete the integration in Eq.(36) above yields:

$$Nu = 0.376\, f(x)\, Pe^{0.5} \quad (37)$$

An energy balance on evaporating two-phase bubble is given by [26] as follows:

$$\frac{dR}{dt} = \frac{(\rho_{dL} - \rho_{dv})}{h_{fg}\rho_{dL}\rho_v}h\Delta T \quad (38)$$

With boundary conditions:

$$t = 0 \qquad R = R_o \quad (39)$$

Equation (38) can be written in dimensionless form as:

$$\frac{dB}{d\tau} = 2(1 - RRD)\frac{Ja\, Fr_o\, Nu}{B\, Pe_o} \quad (40)$$

With boundary conditions:

$$\tau = 0 \qquad B = 1 \quad (41)$$

Where:

$$B = \frac{R}{R_o}, \tau = \frac{gt}{u_o}, Fr = \frac{u^2}{2Rg}, Fr_o = \frac{u_o^2}{2R_o g}, Re = \frac{2\rho_c uR}{\mu_c}, Re_o = \frac{2\rho_c u_o R_o}{\mu_c},$$

$$Pe = \Pr Re\,,\ Pe_o = \Pr Re_o, Nu = \frac{2hR}{k}, RRC = \frac{\rho_{dL}}{\rho_c}, RRD = \frac{\rho_v}{\rho_{dL}}, Ja = \frac{\rho_c Cp\Delta T}{\rho_v h_{fg}} = \frac{St}{RRC\,.RRD}, St = \frac{Cp\Delta T}{h_{fg}}, U = \frac{u}{u_o}$$

Substituting Eq.(37) into Eq.(41) result :

$$\frac{dB}{d\tau} = 0.752\, f(x)(1 - RRD)\frac{Ja\, Fr_o U^{\frac{1}{2}}}{B^{\frac{1}{2}} Pe_o^{\frac{1}{2}}} \quad (42)$$

Where U is a dimensionless two-phase bubble rise velocity .

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Farqid [23] found the two-phase bubble rising velocity as follows:

$$U = \left[\frac{1.333\ g\,(\rho_c - \rho_{dL}) R_o^{\frac{5}{8}} B^{\frac{5}{8}}}{Ja\ u_o^{\frac{4}{8}} \rho_c^{\frac{1}{8}} \left(11.372\, Pr^{\frac{1}{2}} \mu_c^{\frac{2}{8}} + 15.258 \left(\frac{r-1}{r}\right)\left(\frac{k}{Cp}\right)^{\frac{2}{8}}\right)}\right]^{\frac{8}{4}} \qquad (43)$$

The vaporization ratio x can found using the relation:

$$x = \left(\frac{R_v}{R}\right)^3 \qquad (44)$$

where R_v is the radius of the vapor bubble growing inside the drop. It can by find using the mass balance of the system as follows:

Total mass = mass of un evaporating liquid + mass of vapor

Mathematically;

$$\rho_{dL} V_o = \rho_{dL} V_{dL} + \rho_{dv} V_{dv} \qquad (45)$$

Where, V_o , V_L *and* V_v are the initial volume of the drop , the volume of un evaporating liquid and the volume of vapor respectively. The volume of un evaporating liquid and the volume of the vapor and the total volume (V) are related to each other as follows:

$$V = V_{dL} + V_{dv} \qquad (46)$$

Combining the two equations (Eq. (45) and Eq. (46)) with some algebra , we get the following relation for the vapor bubble radius:

$$R_v = \left[\left(\frac{\rho_{dL}}{(\rho_{dL} - \rho_{dv})}\right)(R^3 - R_o^3)\right]^{\frac{1}{3}} \qquad (47)$$

Substituting Eq.(47) into Eq.(44) with $\left(\frac{\rho_{dL}}{\rho_{dL} - \rho_v}\right) \approx 1$, yields:

$$x = \left(1 - \frac{1}{B}\right)^3 \qquad (48)$$

Now, Eq.(43) can be solved analytically using Eq.(29), and Eq.(43) and the relation described by Eq.(44) , results :

$$B = \left[1 + \left(0.8059(1 - RRD)\,\frac{Z\ Ja\ Fr_o\,\tau}{Pe_o^{\frac{1}{2}}}\right)\right]^{1.1429} \qquad (49)$$

Where :

$$Z = \left[\frac{1.333\ g\,(\rho_c - \rho_{dL}) R_o^{\frac{5}{8}}}{Ja\ u_o^{\frac{4}{8}} \rho_c^{\frac{1}{8}} \left(11.372\, Pr^{\frac{1}{2}} \mu_c^{\frac{2}{8}} + 15.258 \left(\frac{r-1}{r}\right)\left(\frac{k}{Cp}\right)^{\frac{2}{8}}\right)}\right]^{\frac{8}{4}} \qquad (50)$$

For β be taken 135^o to obtain the maximum heat transfer coefficient, Eq.(37) (derived already) becomes as follows:

$$Nu = 0.2992\ Pe^{0.5} \qquad (51)$$

3. Results and Discussion

The heat transfer coefficient during vaporization process is generally lower than the case of condensation, this is because of in the former the substantial heat transfer is presumed to occur only through the rear part of the vaporizing drop where unvaporized liquid accumulates. This is because of the internal heat transfer resistance at the frontal part is large due to the low thermal conductivity of the vapour [3]. On the other hand, in condensation process the heat transfer is presumed to be occurred principally through the frontal part of bubble, which is occupied by a thin film of condensate or a number of thin lens-shaped drops. The fraction of heat transfer through the rear part of bubble surface where the condensate cumulates is small , because the thermal resistance in the condensate and the continuous phase in that part are larger than that in the frontal part [24].

Figures (1,2) show the variation of radius ratio with dimensionless time for various operational parameters. The model is verified by comparing with relevant theoretical published results. To do so, Figs. (1,2) demonstrate a comparison between the present model results (Eq.(50)] and both Batya's et al. [25] and Manhood [18] theoretical results. A satisfactory agrees can be shown among the three models. It is clear that the deviation of the present model's results is relatively large for the case of [25] results. For all cases, the trend of the time history of the growth is agreed well. The relatively large deviation could be resulted by the assumptions that are used in the deviation of each model, which is normally acceptable to simplify the complex heat transfer phenomena.

Figure (3) represents a comparison between the present work equation (42) with x=0.05 and 0.1 respectively and different theories. It is obvious that the present solution, when x=0.05 is agreed very well with [3] results. This indicates that the model could me more suitable for application at earlier stage of evaporation when the process is highly transient.

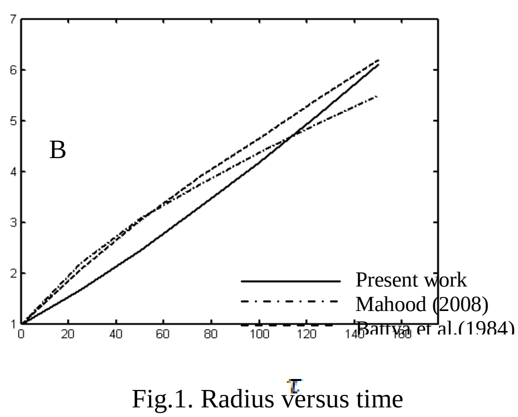

Fig.1. Radius versus time

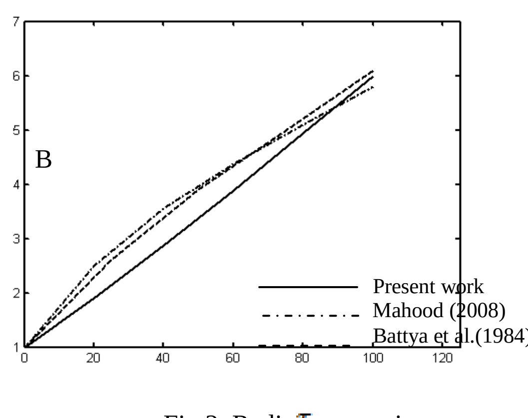

Fig.2. Radius versus time

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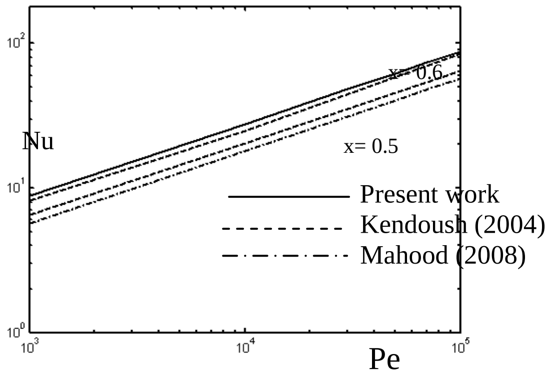

Fig3. Nu versus Pe

IV. Conclusion

According to the results the following conclusions can be made:

- The convective heat transfer coefficient increase with increase vapor mass fraction x. i.e. the convective heat transfer coefficient increase with increase two-phase bubble diameter.

-The concentric spheres model can be used successfully to predict the convective heat transfer coefficient and two-phase bubble growth rate for an evaporation volatile liquid drop in an immiscible media.

Nomenclature

h	heat transfer coefficient (kW/$m^2.h.K$)
k	thermal conductivity (kW/m.K)
M	constant, equation (23)
Nu	Nusselt number (----)
Pe	Peclet number (----)
q	heat flux (kW/$m^2 h$)
q_θ	local heat flux (kW/$m^2 h$)
R	radius of drop (m)
Re	Reynolds number (----)
r	radial coordinate (-----)
T	temperature (K)
T_0	initial drop temperature (K
t	time (s)
U	velocity (m/s)
V_r	radial velocity (m/s)
V_θ	tangential velocity (m/s)
x	vaporization ratio (----)
y	radial distance from drop (m)

Greek symbols

$\propto$	thermal diffusivity (m^2/s)
β	opening half-angle of vapor phase (degrees)
δ	thermal boundary layer thickness (m)
Φ	variable, equation (21)
ψ	variable, equation (19)

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