

# Theories for Analysis of Composite Laminates : A Review

Dhiraj P.Kapoor<sup>1</sup>, Ranjeet S.Mohod<sup>2</sup>

<sup>1</sup>Assistant Professor, Government College of Engineering, Chandrapur, Maharashtra, India

<sup>2</sup>Assistant Professor, Government College of Engineering, Amaravati, Maharashtra, India

**Abstract :** *Laminated composite straight and curved beams are frequently used in various engineering applications. This work attempts to review most of the research done in recent years (1989–2012) on the vibration analysis of composite beams. This review is conducted with emphasis given to the theory being applied (thin, thick, layer wise), methods for solving equations (finite element analysis, differential transform and others) experimental methods, smart beams (piezoelectric or shape memory), complicating effects in both material and structure (visco-elastic, rotating, tip mass and others) and other areas that have been considered in research. A simple classic and shear deformation model would be explained that can be used for beams with any laminate. In the present paper, basic terminology of laminated composite plates is discussed. In aerospace, military and automotive industries laminated composite plate structures find numerous applications. As the material is weak in shear due to its low shear modulus compared to extensional rigidity the role of transverse shear is very important in composites. Hence, an accurate understanding of their structural behavior is required, such deflections and stresses. From the present Literature Review the effect of bending, buckling, thermal & hydrothermal on composite plates discussed and different theories like Classical Plate Theory, First Order Shear Deformation Theory, and Higher order Shear Deformation Theory etc. for analysis of composite plate are mentioned.*

**Keywords :** *composite, Beam, piezoelectric , shape memory, Classical Plate Theory, First Order Shear Deformation Theory*

## Introduction

Laminated composite beams, plates and shells have been used in extensive applications of many engineering fields in recent decades. Structures composed of composite materials offer lower weight and higher strength and stiffness than those composed of most metallic materials. These advantages coupled with the ability to tailor designs for specific purposes, gave them a competitive edge when compared with normal engineering materials and led to their extensive use. Composite beams, plates and/or shell components have found increasing use in recent aerospace, submarine automotive structures.

Laminated composite materials are increasingly being used in a large variety of structures including aerospace, marine and civil infrastructure owing to the many advantages they offer: flexibility to vary fiber orientation, for lower weight high strength/stiffness, good fatigue response characteristics, material and stacking pattern, resistance to electrochemical corrosion, and other superior material properties of composites.

One of the important problems in composite structural design is the vibration analysis of composite beams. Composite beams act as lightweight load carrying structures in diverse application. Literature on composite beam research can be found in many conferences and journals. Kapania and Raciti [1] made a review on advances in analysis of laminated beams and plate's vibration and wave propagation in 1989. Rosen [2] reviewed the research on static, dynamic, and stability analysis of pre-twisted rods and beams in 1991. Chidamparam and Leissa [3] reviewed the published literature on the vibrations of curved bars, beams, rings and arches of arbitrary shape which lie in a plane in 1993. Also a book [4] was dedicated to vibration of composite beams, plates and shells. This article focuses on the last two decades of research (1989–2012) done on the vibration analysis of composite beams. The literature is reviewed while focusing on various aspects of research. We will first review the various beam theories that are being used in research in recent years. These include thin (or classical), thick(or shear deformation), and layer wise beam theories. Then different methods for solving equations of motion such as transfer matrix method, finite element method and others would be reviewed. Another aspect of research will be use of smart materials, which include electro-rheological fluids, piezoelectric sensors and actuators, and shape memory materials. Complicating effect will be the final category that will be addressed. This will include visco-elastic effects, added mass, rotating beam, beams with damages and so on.

## 2. Beam theories

Beams are generally three dimensional (3D) bodies bounded by four, relatively close surfaces. The 3D equations of elasticity are unnecessarily complicated when written for a beam. Researchers simplify such equations by making certain assumptions for particular applications. Almost all beam theories reduce the 3D elasticity problem into a one dimensional (1D) problem.

There are two issues typically treated for 1D analysis of beams. The first problem is the issue of coupling and how to include the various couplings (stretching bending, bending twisting and others) that are ignored in reducing 3D equations to 1D. A suitable approach for inclusion of coupling parameters is to redefine stiffness parameters such that it includes other couplings. Coupling problem can be solved by using equivalent stiffness parameters instead of normal definition of  $A_{11}$ ,  $B_{11}$ , and  $D_{11}$ . That is why different definitions of stiffness parameters are presented first.

## 2.1. Stiffness parameters

Kaw [5] suggested defining the stiffness parameters based on the flexibility matrix (the ABD matrix inverse). Rios and Chan [6] proposed another formulation based on ABD inverse matrix. Ecsedi and Dluhi [7] analyzed the static and dynamic behavior of non-homogeneous curved beams and closed rings. The modulus was treated as a function in cylindrical coordinates in place of ABD terms. Some other researchers [8–11] treated the problem of laminated composite beams to that of isotropic homogeneous beams with effective bending (EI), torsional (GJ), and axial stiffness parameters (EA). As per author's knowledge in analysis of generally laminated beams none of these models were accurate. Hajianmaleki and Qatu [12] showed that using equivalent modulus of elasticity of each lamina, one can get accurate results for static and dynamic analyses of generally laminated beams with any kind of coupling. The equivalent modulus of elasticity of each lamina is found using [12,13].

## 2.2. Effect of shear deformation

The inclusion of shear deformation in the analysis of beams was first made by Timoshenko [17]. Hence, theories considering shear deformation are called as Timoshenko beam theories. In this regard, the beam theories are classified based on the order of polynomial for approximation of displacements through the thickness.

### 2.2.1. Classical beam theory

If the beam thickness is less than  $1/20$  of the wavelength of the deformation mode, a classical beam theory (CBT) or Euler Bernoulli (EB) beam theory where shear deformation and rotary inertia are negligible, is generally acceptable for lower frequencies determination. Qatu [19] and Qatu and Elsharkawi [20] used CBT to study vibration of straight and curved cross-ply laminated beams. They used Ritz method for solving the equations of motion. To solve the vibration of a cross-ply laminated composite driveshaft with an intermediate joint Qatu and Iqbal [21] used CBT. The effect of coupling between bending and torsional deformations on vibrations of composite EB beams from a wave vibration standpoint was studied by Mei [22]. The

torsional mode was found affected by the material coupling only at low frequencies. The flexural modes were found to be affected by material coupling over the entire frequency band. Mei [23] also studied the local wave transmission and reflection characteristics at various discontinuities on composite beams. Gunda et al. [24] investigated large amplitude vibration of laminated composite beams with symmetric and asymmetric layup orientations and axially immovable ends. They used CBT and solved the equations by Rayleigh–Ritz method. For the membrane stretching action of the beam Geometric nonlinearity of von-Karman type was considered.

## Conclusion

Based on the requirement of problem under consideration many of the researchers have applied various theories for the solution of problems at hand. Based on the selection of polynomial displacement formulations the correctness of the solution is mostly affected and thereby based on the error encountered researchers are claiming that particular theory is converging more precisely to the classical solution. For through thickness stresses more appropriate are shear deformation theories. Many of the layer wise theories have also yielded good result in agreement with the classical solutions.

## References

- i. Kapania RK, Raciti S. *Recent advances in analysis of laminated beams and plates, part II: vibration and wave propagation*. AIAA J 1989;27(7):935–46.
- ii. Rosen A. *Structural and dynamic behavior of pretwisted rods and beams*. Appl Mech Rev 1991;44:483–515.
- iii. Chidamparam P, Leissa AW. *Vibrations of planar curved beams, rings, and arches*. Appl Mech Rev 1993;46(9):467–84.
- iv. Qatu MS. *Vibration of laminated shells and plates*. Netherlands: Elsevier Academic Press; 2004.
- v. Kaw AK. *Mechanics of composite materials*. Boca Raton (FL): CRC Press; 2005.
- vi. Rios G, Chan WS. *A unified analysis of stiffened reinforced composite beams*. In: *Proceedings of 25th ASC conference, Dayton (OH), 20–23 September 2009*.
- vii. Ecsedi I, Dluhi K. *A linear model for the static and dynamic analysis of nonhomogeneous curved beams*. Appl Math Model 2005;29:1211–31.
- viii. Banerjee JR. *Frequency equation and mode shape formulae for composite Timoshenko beams*. Compos Struct 2001;51:381–8.
- ix. Banerjee JR. *Explicit analytical expressions for frequency equation and mode shapes of composite beams*. Int J Solid Struct 2001;38:2415–26.
- x. Li J, Shen R, Hua H, Jin J. *Bending–torsional coupled dynamic response of axially loaded composite Timoshenko thin-walled beam with closed crosssection*. Compos Struct 2004; 23–35.

- xi. El Bikri K, El Bekkaye M, Benamar R. Geometrically nonlinear free vibrations of laminated composite beams: an effective formulation. *Appl Mech Mater* 2011;105–107:1681–4.
- xii. Hajianmaleki M, Qatu MS. Mechanics of composite beams. In: Tesinova P, editor. *Advances in composite materials-analysis of naturally and man-made materials*. Croatia: InTech Publishing Company; 2011. p. 527–46.
- xiii. Vinson JR, Sierakowski RL. *The behavior of structures composed of composite materials*. Netherlands: Kluwer Academic Publishers; 2002.
- xiv. Hajianmaleki M, Qatu MS. Static and vibration analyses of thick generally laminated deep curved beams with different boundary conditions. *Compos Part B: Eng* 2012;43(4):1767–75.
- xv. Qatu MS. Theories and analyses of thin and moderately thick laminated composite curved beams. *Int J Solid Struct* 1993;30(20):2743–56.
- xvi. Leissa AW, Qatu MS. *Vibration of continuous systems*. USA: McGraw Hill; 2011.
- xvii. Timoshenko SP. On the correction for shear of the differential equation for transverse vibrations of prismatic beams. *Philos Mag* 1921;6(41):744–6.
- xviii. Khdir AA, Reddy JN. Free vibration of cross-ply laminated beams with arbitrary boundary conditions. *Int J Eng Sci* 1994;32(12):1971–80.
- xix. Qatu MS. In-plane vibration of slightly curved laminated composite beams. *J Sound Vib* 1992;159(2):327–38.
- xx. Qatu MS, Elsharkawi AA. Vibration of laminated composite arches with deep curvature and arbitrary boundaries. *Comput Struct* 1993;47(2):305–11.
- xxi. Qatu MS, Iqbal J. Transverse vibration of a two-segment cross-ply composite shafts with a lumped mass. *Compos Struct* 2010;92(5):1126–31.
- xxii. Mei C. Effect of material coupling on wave vibration of composite Euler– Bernoulli beam structures. *J Sound Vib* 2005;288:177–93.
- xxiii. Mei C. Free and forced wave vibration analysis of axially loaded materially coupled composite timoshenko beam structures. *J Vib Acoust* 2005;127:519–29.
- xxiv. Gunda JB, Gupta RK, Janardhan GR, Rao GV. Large amplitude vibration analysis of composite beams: simple closed-form solutions. *Compos Struct* 2011;93:870–9.
- xxv. Hajianmaleki M, Qatu MS. Transverse vibration analysis of generally laminated two-segment composite shafts with a lumped mass using generalized differential quadrature. *J Vib Control* 2012.