# A Comparative Study on Numerical Buckling Analysis for a Cantilever Steel Pipe Column under Combined Loads

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**Abstract**：*Buckling of a cantilever steel pipe column under combined loads was studied through linear and nonlinear numerical analysis method. Firstly, linear buckling analysis of the cantilever column with linear-elastic material was used to select appropriate element type and element size for this problem. Then linear buckling and nonlinear buckling analyses for an imperfect cantilever column under different horizontal loads or displacements in the context of elasticity were performed to verify the ability of the linear buckling analysis to include large geometric changes. Thirdly, nonlinear analyses were carried out to examine the effect of plastification of material on the buckling limit loads for the imperfect cantilever column. Through these comparative studies, some aspects concerned with the numerical buckling analysis of structures such as columns were clarified.*

**Keywords:** eigenvalue buckling, nonlinear buckling, cantilever column, finite element analysis.

## I. Introduction

Stability of a structure subjected to external loads means it could return from disturbed shape to its original configuration after the applied minor disturbance removed (Timoshenko & Gere 1985). Conversely a structure is under unstable state if it has to find a new equilibrium with a large change in configuration or keep on deviating from its original equilibrium shape along with external loads decreasing, after being disturbed. The process by which a structure loses stability refers to as buckling. Buckling loads are defined as the external loads corresponding to the critical state of the structure between stability and instability. There are two types of analyses concerned with determining buckling loads of a structure, namely linear buckling analysis and nonlinear buckling analysis, depending on geometry of the structure, mechanical behavior of material comprised the structure and applied loads. Linear buckling analysis, also named as eigenvalue buckling analysis, is generally used to analyze the stability problem of an ideal linear elastic structure. The lowest eigen value obtained by analytical or numerical method is the buckling load. However, from the engineering view, imperfections, such as geometrical, structural and load imperfection, are unavoidable for a structure, which affect the relationship between the applied loads and deflections (Prabu et al. 2010). Nonlinear buckling analysis, taking the numerical method as a tool, is recommended to include the influence of imperfections on stability of a structure (Silva 2006, Novoselac et al. 2012).

The commercial finite element software ABAQUS has incorporated a linear perturbation procedure to predict eigenvalue buckling loads, which is relative to the base state of a structure. For nonlinear buckling problems the Riks analysis technique, based on "arch length" approach, is provided by ABAQUS. It is generally used to predict unstable, geometrically nonlinear collapse of a structure consisted of elasto-plastic material, where the load-displacement response perhaps shows negative stiffness.

In this paper, some efforts are made to compare eigenvalue and nonlinear buckling analysis of a cantilever column under combined loads using ABAQUS. Firstly, the element size and element type were examined based on eigenvalue buckling analysis and the recommendation of them was made. Secondly, eigenvalue and nonlinear buckling analyses were performed for structures with large change of geometry to verify that the influence of geometrical nonlinearity on buckling load was included in eigenvalue buckling analysis. Finally, the effect of plasticity of material on the buckling limit load for the imperfect cantilever column built by structural element and solid element was studied through nonlinear buckling analysis.

The cantilever pipe column is made of Q235 steel with length 40m, outside diameter 800mm and thickness 10mm. The Young's modulus of this kind of steel is 206GPa and Poisson's ratio is 0.3. The critical buckling stress is 200MPa and yielding stress is 240MPa. The column belongs to a slender one and its critical buckling load is 615.41kN obtained from Euler formula.

## II. Selection of element type and element size

The steel pipe column can be discretized by using structural element or solid element for finite element analysis. In order to compare the influence of numerical model with different element type and element size on finite element analysis for stability problem of the cantilever steel pipe column, seven element types and four element sizes were selected to discretize the column, which were shown in Table 1. Except a full-size model established for numerical model with beam element, half-size models were applied for numerical models using the other element types, considering that the buckling deformation of column is symmetry about column's longitudinal plane. This symmetry was realized through restraining displacement along the normal direction of longitudinal symmetry plane. Fifty elements were adopted along circumference of column for full-size model (circumferential element size is about 25mm) and the radial element size is the thickness of column. Eigenvalue buckling loads of linear elastic cantilever column under axial load, obtained by finite element analysis, were presented in Table 1. It can be seen that the longitudinal element size hardly has effect on the buckling loads, but the element type has. Buckling loads obtained by using element B23, C3D20R and CCL24R are in accordance with analytical solution closely. The numerical results from shell element are about 3.8% greater than the analytical one, however, the element C3D8R give conservative

results with a minor decrease of 0.25 per cent.

Table 1  Eigenvalue buckling loads of cantilever column (kN)

| longitudinal element size (mm) | 500 | 100 | 4 | 2.5 |
|---|---|---|---|---|
| Beam element B23 | 615.08 | 615.07 | 615.02 | 615.48 |
| Shell element S4R | 638.34 | 638.21 | 638.81 | 637.89 |
| Solid shell element SC8R | 616.92 | 615.08 | 614.82 | 614.74 |
| Solid element C3D8R | 614.08 | 613.92 | 613.92 | 613.92 |
| Solid element C3D20R | 615.06 | 615.06 | 615.06 | 615.06 |
| Cylindrical solid element CCL24R | 615.06 | 615.06 | 615.06 | 615.06 |
| Cylindrical solid element CCL12 | 617.80 | 617.80 | 617.80 | 617.80 |

The effect of circumferential element size on eigenvalue buckling loads was examined by model established with first-order solid element C3D8R. The number of element along circumference of column is 40, 30, and 20 respectively, while the longitudinal size of element keeps 100mm. The numerical results from elements with these three kinds of sizes, which are 613.18kN, 612.22kN, and 608.70kN, show that buckling loads decrease with increase of circumferential element size.

From the comparison of accuracy of buckling loads, together with considering of calculation cost, the beam element is recommended for simple structures. However, for complex structures involving interaction, the beam element cannot deal with it. Numerical model, therefore, established with first-order solid element C3D8R and discretized with relatively coarse mesh should be selected.

### III.  Comparison of analysis method for cantilever column with larger displacement on top

Cantilever column often used as an equivalent to pile foundation in buckling analysis, of which the length depends on geometry of pile, mechanical behavior of pile and subsoil. The scenarios that piles are subjected to horizontal load as well as axial load are common in offshore engineering. Khodair and Hassiotis (2006) once analyzed the buckling load of pile subjected to horizontal load or displacement on the top of pile using linear buckling analysis method provided by ABAQUS. A conclusion that the buckling load decreases with increase of horizontal load or displacement was drawn through the study. However, Kerciku and Bhattacharya (2007) thought the buckling load of piles is a function of flexural stiffness, length and boundary conditions and is independent of lateral perturbations, and the buckling load obtained by linear buckling analysis for this kind of problem was meaningless. Kerciku and Bhattacharya confused disturb in linear perturbation step with displacement occurred during the process of forming base state. The linear buckling analysis method does take account of the effect of displacement existing in base state on the buckling load. In order to clarify this point, linear and nonlinear buckling analyses were performed for cantilever column with larger displacement on top. Beam element was used to discretize the steel pipe column and linear elastic material with the same modulus of elasticity and Poisson's ratio as the above was assumed. Two types of general loads, horizontal concentrate load and horizontal displacement, were applied to the top of the

column solely in the step of forming base state, aiming to ascertain options for specifying displacement conditions in linear perturbation step. Table 2 shows the relationship between horizontal loads and displacements obtained from finite element analysis.

Table 2 Relationship between horizontal concentrate load and horizontal displacement

| Horizontal load $H$ (kN) | 1.0 | 5.0 | 10.0 | 50.0 | 100.0 |
|---|---|---|---|---|---|
| Horizontal displacement $h$ (m) | 0.054 | 0.267 | 0.535 | 2.662 | 5.349 |
| $H / L$ (%) | 0.134 | 0.669 | 1.338 | 6.656 | 13.372 |

Eigenvalue buckling loads for steel cantilever column under different horizontal loads or horizontal displacements are presented in Table 3. Obviously, the horizontal displacement as well as the horizontal load has effect on decreasing eigenvalue buckling load. One can also find from Table 3 that the decrease of buckling load with increase of horizontal displacement or load is small, for example, the decrease is only 7.13% even when the horizontal displacement being up to 13.37% of length of column. Figure 1 shows the deformation of column in base state and the first eigenvalue buckling mode, which further testify that horizontal displacement is truly included in base state and affects the eigenvalue even it has the same deformation mode as perturbation for eigenvalue extraction.

Table 3  Eigenvalue buckling loads for steel cantilever column

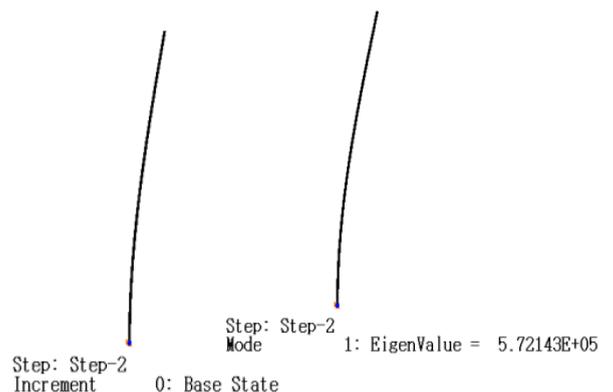| Horizontal load $H$ (kN) | 0.0 | 1.0 | 5.0 | 10.0 | 50.0 | 100.0 |
|---|---|---|---|---|---|---|
| Eigenvalue buckling load $P_{cr}$ (kN) | 615.08 | 615.07 | 614.95 | 614.59 | 603.38 | 571.23 |
| Horizontal displacement $h$ (m) | 0.0 | 0.054 | 0.267 | 0.535 | 2.662 | 5.349 |
| Eigenvalue buckling load $P_{cr}$ (kN) | 615.08 | 615.07 | 614.95 | 614.59 | 603.38 | 571.14 |



Figure 1 Deformation for the base state and the first order mode of column under horizontal displacement 5.3487m

The forgoing cantilever column subjected to horizontal load or displacement belongs to imperfect structures from the viewpoint of buckling analysis. Nonlinear buckling analysis with Riks technique is recommended to predict buckling load of this problem, which is regarded as a more accurate method than the eigenvalue buckling analysis. The relationships between vertical load and horizontal displacement predicted by Riks method for cantilever column under different horizontal loads or

displacements are presented in Figure 2, of which the abscissa is horizontal displacement of the top of the column with the horizontal displacement caused by horizontal preload is deducted, and the vertical axis is the vertical load applied on the top of column. As can be seen, each load-displacement path makes eigenvalue buckling load of ideal linear elastic cantilever column, $P_{cr}$ as its asymptotic line. The lower the horizontal load is, the faster the load displacement path converges to $P_{cr}$. As far as this type of load-displacement path without limit load point is concerned, the buckling limit load can be determined by virtue of "double tangent" method used in obtaining limit bearing capacity of footings (Nawghare et al. 2010). As seen in Figure 3, two tangents were made to the first and the last straight part of load-displacement path, and then vertical load corresponding to intersection of these two tangents is the buckling limit load. Buckling limit loads from nonlinear buckling analysis and eigenvalue buckling loads from linear buckling analysis are compared in Table 4, from which one can find that the linear buckling analysis method overestimates the buckling load to some degree and the overestimated value increases with the increase of horizontal load. This is because, for linear buckling analysis provided by ABAQUS, the nonlinearity from geometrical change caused by preload or pre-displacement can only be accounted of in the process of forming base state, while it is not considered continuously for eigenvalue extraction in successive linear perturbation step. From this comparison, one can conclude that eigenvalue buckling analysis including large change of geometry can be used for rough behavior study of buckling of structures with less computation cost.
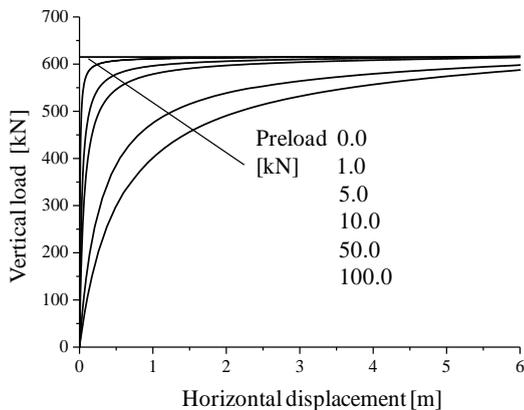


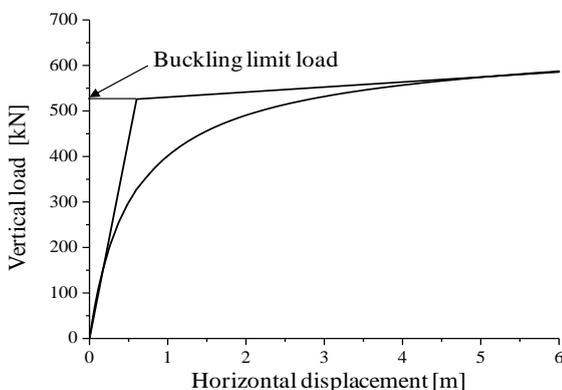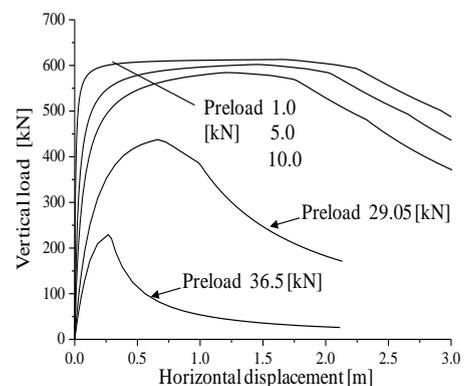Figure 2 Load displacement path predicted by nonlinear buckling analysis



Figure 3 Double-tangent method

Table 4 Eigenvalue loads and buckling limit loads

| Horizontal load $H$ (kN) | Eigenvalue buckling load $P_{cr}$ (kN) | Buckling load $P_u$ (kN) | $(P_{cr}-P_u) / P_u$ (%) |
|---|---|---|---|
| 0 | 615.08 | - | - |
| 1 | 615.07 | 608.31 | 1.11 |
| 5 | 614.95 | 602.41 | 2.08 |
| 10 | 614.59 | 595.77 | 3.16 |
| 50 | 603.38 | 556.58 | 8.41 |
| 100 | 572.23 | 525.95 | 8.80 |

### IV  Buckling limit load considering plastic behaviour of material

If the steel of column is regard as linear elasto-perfectly plastic material, the cantilever column begins to yield when the horizontal load applied on the top of column is 29.05kN, and the corresponding horizontal displacement is 1.55m. The cantilever column is just in fully plastic state when a horizontal load, 37.73kN applied, and a value of 2.09m of horizontal displacement occurs at the top of column. Then the buckling limit load should be zero under this fully plastic condition, from the view point of stability. In this section, nonlinear buckling (Riks) analyses were carried out for numerical cantilever model established by beam element, B23 and first-order solid element, C3D8R, to examine the effect of element type on buckling load when plasticity of material considered. It can be seen from Figure 4 that all the load-displacement paths with different horizontal preloads or pre-displacements show a tendency of decline after reaching peak value. The peak value is defined as the buckling limit load and its decrease with increase of horizontal preloads or pre-displacements reflects the effect of plasticity on stability problems, for example, the peak value is less than 5kPa when the preload is 37.8kN, which is beyond the fully plastic load limit, 37.73kN. One can also find the load-displacement paths of first-order solid element are different from that of beam element, which predicts lower buckling limit loads than beam element, compared in Table 5. The difference results from the effect of shear deformation which is not considered by beam element, B23 based on Euler-Bernoulli beam theory. Then the conclusion is solid element could predict a more realistic buckling load and should be recommended for stability problems with plasticity considered.
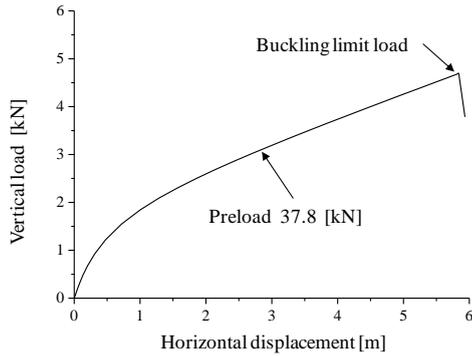
(a) Results of beam element B23
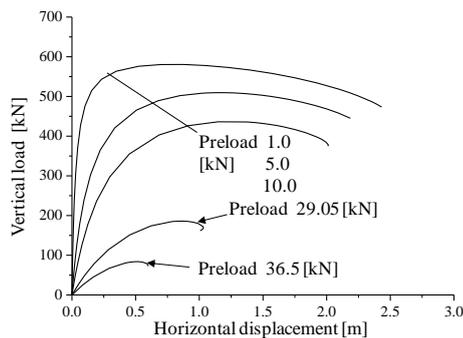


(b) Results of first-order solid element C3D8R
Figure 4 Load displacement path predicted by nonlinear buckling analysis considering plasticity

Table 5 Buckling limit loads considering plasticity of material

| Horizontal preload H | Buckling load of beam element $P_{u1}$ | Buckling load of solid element $P_{u2}$ | $P_{u1}$- $P_{u2}$ |
|---|---|---|---|
| 1 | 613 | 580.5 | 32.5 |
| 5 | 602.14 | 509.56 | 92.58 |
| 10 | 584.22 | 436.6 | 147.62 |
| 29.05 | 437.53 | 185.96 | 251.57 |
| 36.5 | 229.6 | 83.88 | 145.72 |

## V. Conclusions

Linear eigenvalue buckling analysis is usually used for fast estimation of critical buckling load for ideal linear-elastic structures. However, structures always have imperfections due to eccentric loads and/or geometry imperfections. In this paper, a comparative study for numerical analysis of stability problem of a cantilever steel pipe column was carried out through linear and nonlinear buckling analysis. It can be concluded that:

Beam element or second-order solid element can give a good prediction of eigenvalue buckling loads; first-order solid element is applicable to wide situations though it slightly underestimates the buckling loads. Element size has little effect on the eigenvalue buckling loads.

Large change of geometry of structures could be included in forming base state in linear eigenvalue buckling analysis, and this linear analysis procedure suits rough behavior study of buckling of structures with less computation cost.

Solid element, other than structural element, is more suitable for nonlinear buckling analysis for imperfect structures to considering the plastification of material.

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