

# Dimensional Synthesis of Four Bar Mechanism Using Genetic Algorithm

S. S. Shete<sup>1</sup>, S. A. Kulkarni<sup>2</sup>

Mechanical Department, SCoE, Pune, Maharashtra, India

Corresponding Email: <sup>1</sup>ssshete.scoe@gmail.com, <sup>2</sup>sakulkarni.scoe@gmail.com

**Abstract:** The dimensional synthesis is done by using Genetic Algorithm to achieve a desired trajectory. Three problems are analyzed having different curvature. The program is authored in MATLAB<sup>®</sup>2010a. The error is seen to be in the permissible prescribed limit. The prototyping of straight line trajectory analysis is also done in ADAMS<sup>®</sup>.

**Keywords:** Dimensional Synthesis, Optimization, GA, Path Generation.

## 1. Introduction

The synthesis of mechanisms is usually employed to project or predict the functional dependence between geometry and the motions of various parts or links in mechanism under consideration. Conventionally the synthesis is classified into type synthesis, number synthesis and dimensional synthesis. The dimensional synthesis means determination of the dimensions of the individual links. Further the path generation is acquired by either with prescribed timing while the angle of each prescribed point of coupler curve are required to be worked out or without prescribed timing while the angle of each prescribed point of coupler curve is given.

The classical graphical and analytical methods for the dimensional synthesis are discussed in Theory of Machines and Mechanisms by Joseph Edward Shigley(1988) which are restricted up to five precision points wherein the coupler curve is defined. In the case of more than five points the Genetic Algorithm (GA) is implemented for dimensional synthesis. The GA is an optimization technique which is based on natural evolution theory and follows the Darwin's principle of survival of fittest. Professor John Holland investigated the concept of GA in mid 60's for explaining the adaptation process of natural system for creating artificial system that works similarly. The GA converts natural evolution to world of computers, to solve optimization problem. Individuals those are weak will die off, and fit individuals will be selected for reproduction.

Cabrera et al. (2002) formulated the GA and applied to optimize the Euclidean distance error function. This error function is the sum of squares of the difference between the desired path point and generated path point. The intention is to reduce the error between desired point and generated point after every iteration so that at the end of last iteration the error should be within prescribed limits. Laribiet al. (2004) formulated fuzzy logic based GA and optimize the error function called as the orientation structural error of fixed link. Acharya et al. (2009) also implemented different evolutionary algorithms viz GA, differential evolution and particle swarm optimization for the synthesis of four bar mechanism. The intention is to compare the efficiency of different algorithms. Cabrera et al. (2010) again implemented an algorithm which is the modification of the

previous algorithm and the name of this algorithm is Malaga University Mechanism Synthesis Algorithm (MUMSA).

Matekar et al. (2012) formulated the modified distance error function based on longitudinal and transverse errors between prescribed path points and obtained path points.

In this paper the GA is formulated consisting of three operators namely reproduction, crossover and mutation and Euclidean distance error function is used for optimization. The algorithm is implemented on straight line generation problem passing through six desired points. The algorithm is implemented in such a way that, after application of every operator the population size is updated; so population size is updated three times in one iteration.

## 2. Formulation of Objective Function

The Fig.1 shows four bar path generating mechanism  $O_a A B O_b$ .

$$\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} P_{x_L} \\ P_{y_L} \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$$

The position of point P ( $P_{x_i}, P_{y_i}$ ) is calculated at each desired point i from above equation and then given as an input to the equation of error function (1) to evaluate the error.

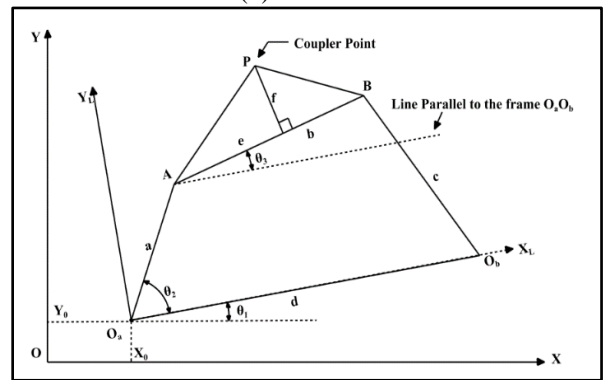


Fig.1 Construction Details of Four bar Linkage

### 2.1 Objective Function

The objective of path synthesis problem is to have a minimum position error and to achieve the objective, dimensions of the four bar linkage i.e. the design variables namely a, b, c, d, e, f links, input angle  $\theta_2$  and  $\theta_1$  angle measured with the fixed link d are determined in such a way that the error between specified point ( $P_{x_{di}}, P_{y_{di}}$ ) and generated point ( $P_{x_i}, P_{y_i}$ ) should be minimized, where  $i = 1, 2, 3, \dots, n$  and n is number of prescribed points. The optimization function is Euclidean distance error function is given below,

$$f(X) = \sum_{i=1}^n [(P_{x_{di}} - P_{x_i})^2 + (P_{y_{di}} - P_{y_i})^2] \quad (1)$$

where, X is the set of design variables to be obtained by optimizing error function.

$X = [a, b, c, d, e, f, \theta_1, \theta_2]$ .

### 2.2 Constraints

The significant equations in the synthesis of mechanism, (1) the objective function and (2) the equality or the inequality constraints which are applied to get optimum solution. The constraints include: Grashof's criterion, input link order, the transmission angle constraint, while the input link should be the smallest and the boundary value of variables.

**Grashof's criterion:**

This criterion is the prevailing condition for the mechanism to make the complete revolution of crank or the input link.  $s + l \leq p + q$ , where,  $s$  and  $l$  are shortest and longest links respectively and  $p$  and  $q$  are the other two links of the mechanism.

**Sequence of input angle constraint:**

$\Delta\theta_2^i = (\theta_2^i - \theta_2^{i-1})$ , where  $i$  is the position of prescribed point.

**Transmission angle constraint:**

In order to get smooth link movements with respect to each other, the force and motion is required to be effectively transferred between them, which prevails that the transmission angle is close to  $90^\circ$ . This is a constraint for the designer, to maintain it in between allowable minimum and maximum values.

$$\mu_{max} = \cos^{-1} \left[ \frac{b^2 - (d+a)^2 + c^2}{2bc} \right] \quad (2)$$

$$\mu_{min} = \cos^{-1} \left[ \frac{b^2 - (d-a)^2 + c^2}{2bc} \right] \quad (3)$$

The actual value of transmission angle at any angle  $\theta_2^i$  is given by,

$$\mu = \cos^{-1} \left[ \frac{b^2 - a^2 - d^2 + c^2 + 2ad \cos \theta_2^i}{2bc} \right] \quad (4)$$

The criterion to be satisfied is  $\mu_{min} \leq \mu \leq \mu_{max}$ .

**The length of Input link must be small:**

The length of input link must be smaller than the lengths of the remaining three links. This is strictly observed in the initial stage so that the constraint is not violated and the penalty is automatically written off.

**Variable bounds:**

Minimum and maximum values of all variables in the design vector are considered in the initial set of random variables and in order to get the value of variable between its minimum and maximum value, the simple generation rule is,  $X = X_{min} + rand * (X_{max} - X_{min})$  (5)

where, rand is a random number generator between 0 and 1.

**Overall Optimization Problem:**

The objective function is the sum of the error function and the penalties added on violation of the constraints as follows;

$$E(X) = f(X) + P1 * C1(X) + P2 * C2(X) + P3 * C3(X) \quad (6)$$

where,  $f(X)$  is the Euclidean distance error function.  $C1(X)$ ,  $C2(X)$ ,  $C3(X)$  are multiplying factors for Grashof's criterion, sequence of input angle and transmission angle criterion respectively and if the criterion is satisfied then corresponding multiplying factor is taken as 0 otherwise 1.  $P1$ ,  $P2$ ,  $P3$  are the penalties which are to be added in the function value if these constraints are not satisfied and these penalties are taken as 1000 as surveyed in various literatures.

**3. Optimization Algorithm**

The flow chart of applied algorithm is given below

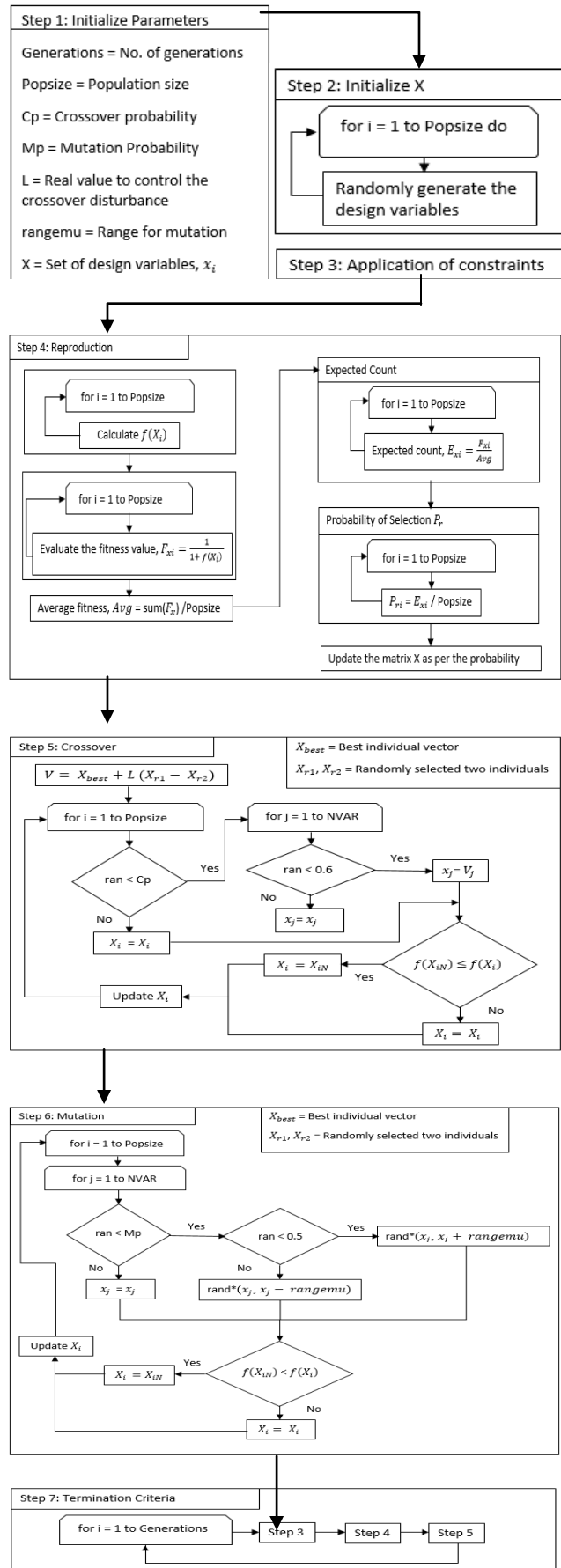


Fig.2 Flow Chart of GA

#### 4. Results and Discussion

The key parameters of an algorithm are efficiency and accuracy. The problem statements given in this paper is to generate a straight line passing through 6 desired points without prescribed timing

##### 4.1 Straight Line Generation Problem

Design variables are,

$$X = [a, b, c, d, e, f, X_0, Y_0, \theta_1, \theta_2^1, \theta_2^2, \dots, \theta_2^6]$$

where,  $a, b, c, d$  = Links of the mechanism

$e, f$  = Construction lines for obtaining the coordinates of coupler point P

$\theta_1$  = Angle made by fixed link  $d$  with X-axis

$\theta_2^i$  = Angle made by input link with fixed link at  $i^{\text{th}}$  position

Target points chosen:

$$C_d^i = [(20,20), (20,25), (20,30), (20,35), (20,40), (20,45)]$$

Limits of the variables:

$$a, b, c, d \in [0,60]; e, f, X_0, Y_0 \in [-60,60];$$

$$\theta_1, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6 \in [0,2\pi].$$

Parameters considered are, Generations= 1000,  $N_p = 100$ ,  $C_p = 0.9$ ,  $M_p = 0.1$ ,  $L = 0.6$ , range = 0.1.

**Table. 1** Synthesized Results for Straight Line Generation Problem

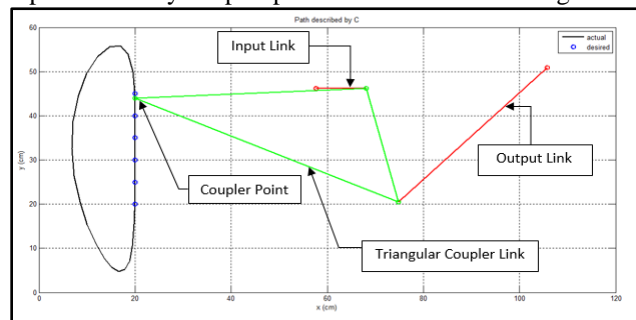
| Design Variable | Value    | Design Variable | Value   |
|-----------------|----------|-----------------|---------|
| a               | 10.4657  | $\theta_2^1$    | 5.33582 |
| b               | 26.4562  | $\theta_2^2$    | 5.52386 |
| c               | 43.3573  | $\theta_2^3$    | 5.69795 |
| d               | 48.3378  | $\theta_2^4$    | 5.86718 |
| e               | -9.9783  | $\theta_2^5$    | 6.03998 |
| f               | -47.2007 | $\theta_2^6$    | 6.22731 |
| $X_0$           | 57.6963  | $\theta_1$      | 0.09769 |
| $Y_0$           | 46.1450  |                 |         |

The synthesized values of all 15 design variables involved are shown in Table. 1. The corresponding values of desired points ( $P_{x_d}, P_{y_d}$ ) and generated points ( $P_x, P_y$ ) and also the error between them is shown in Table. 2.

**Table. 2** Actual Points Traced by Coupler Point for Straight Line Generation Problem

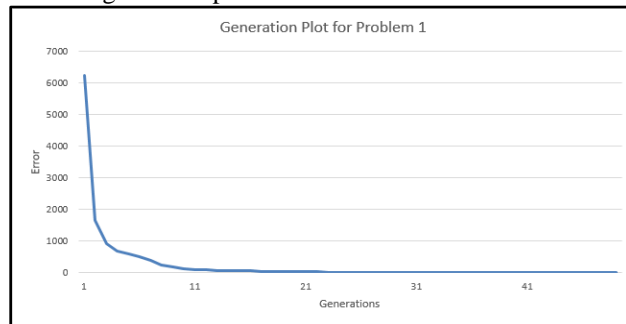
| Sr. No. | $P_{x_d}$ | $P_x$    | $(P_{x_d} - P_x)^2$ | $P_{y_d}$ | $P_y$    | $(P_{y_d} - P_y)^2$ | $(P_{x_d} - P_x)^2 + (P_{y_d} - P_y)^2$ |
|---------|-----------|----------|---------------------|-----------|----------|---------------------|---|
| 1       | 20.00     | 19.96281 | 0.001383            | 20.00     | 19.99645 | 1.2602E-05          | 1.40E-03                                |
| 2       | 20.00     | 20.00857 | 7.34E-05            | 25.00     | 25.00007 | 4.761E-09           | 7.34E-05                                |
| 3       | 20.00     | 20.02686 | 0.000722            | 30.00     | 29.99997 | 8.41E-10            | 7.22E-04                                |
| 4       | 20.00     | 20.0388  | 0.001505            | 35.00     | 34.99822 | 3.1541E-06          | 1.51E-03                                |
| 5       | 20.00     | 20.02997 | 0.000898            | 40.00     | 39.99919 | 6.561E-07           | 8.99E-04                                |
| 6       | 20.00     | 19.93483 | 0.004248            | 45.00     | 45.00122 | 1.4859E-06          | 4.25E-03                                |

The path traced by coupler point is shown below in Fig. 3.



**Fig. 3** Path Traced by Coupler Point for Straight Line Generation Problem

The objective is to generate a straight line trajectory from a coupler point of the four bar mechanism. The coupler point is the point on a triangular coupler link. The four bar mechanism contains four links namely input link, triangular coupler link, output link and fixed link as shown in Fig. 3. The coordinates (20,20) and (20,45) are the starting and end points of the desired straight line. There are 6 desired points through which the coupler point should pass. The generated coupler curve is shown in Fig. 3 and it is in elliptical shape but the right portion of generated elliptical curve is straight line and this straight line is passing through all desired points. The total error between desired and generated points is 0.0088.



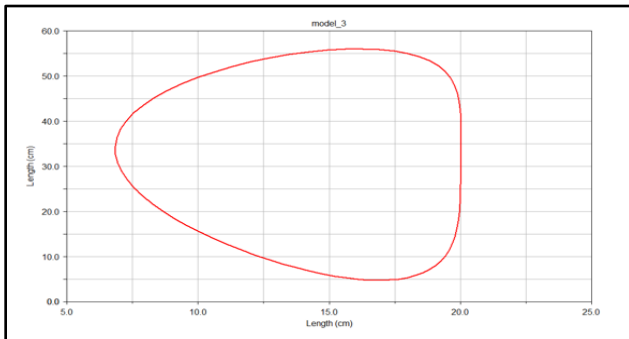
**Fig. 4** Plot of Error Vs Generations Count for Straight Line Generation Problem

The GA implemented in MATLAB® 2010a gives off the graph of error verses generations count is plotted as shown in Fig. 4. Initially the structural error is more than 6000 and it is reduced up to 1 after 100<sup>th</sup> generation. 1000 generations carried out and at the end of 1000<sup>th</sup> generation the structural error reduces to 0.0088. The error in X and Y direction at each position is calculated and plotted as shown in Fig. 5.



**Fig. 5** Error in X and Y Direction for Straight Line Generation Problem

**Simulation of mechanism:** The prototyping of mechanism is done in ADAMS<sup>®</sup> to co-relate the simulation results with generated curve. The dimensions obtained from the synthesis of straight line analysis problem are used to form a mechanism in ADAMS<sup>®</sup>. The mechanism consists of input link, coupler link, and output link. The circular motion of input link is converted into elliptical motion of coupler point of triangular coupler link. If the input link is rotated in anticlockwise direction then in the return motion, coupler point gives straight line and it is analogous with the generated coupler curve in MATLAB<sup>®</sup> 2010a. The coupler curve traced in ADAMS is shown in Fig. 6.



**Fig. 6** Coupler Curve traced in ADAMS<sup>®</sup>

## 5. Conclusion and Future Scope

In straight line generation problem, the error is reduced to 0.008 which is because population size is updated three times in each iteration. The obtained coupler curve is analogous with the desired curve. The GA allows more number of design variables as compared with analytical and graphical methods.

## References

- i. Joseph Edward Shigley (1988), "Theory of Machines and Mechanisms", The McGraw Hill Company.
- ii. J.A. Cabrera, A. Simon, M. Prado (2002), "Optimal synthesis of mechanisms with GAs", *Mechanism and Machine Theory*, vol. 37, pp. 1165–1177.
- iii. M. A. Laribi, A. Mlika, L. Romdhane, S. Zegloul, (2004), "A combine GA-fuzzy logic method (GA-FL) in mechanisms synthesis", *Mechanism and Machine Theory*, vol. 39, pp. 717–735.
- iv. S.K. Acharyya, M. Mandal, (2009), "Performance of EAs for four-bar linkage synthesis", *Mechanism and Machine Theory*, vol. 44, pp. 1784–1794.
- v. J.A. Cabrera, A. Ortiz, F. Nadal, J.J. Castillo, (2010), "An evolutionary algorithm for path synthesis of mechanisms", *Mechanism and Machine Theory*, vol. 46, pp. 127–141.
- vi. Sanjay B. Matekar, Gunesh R. Gogate, (2012), "Optimum synthesis of path generating four-bar mechanisms using differential evolution and a modified error function", *Mechanism and Machine Theory*, vol. 52, pp. 158–179.
- vii. Han Jianyou, Qian Weixiang, Zhao Huishe, (2007), "Study on synthesis method of  $\lambda$  – formed four bar linkages approximating a straight line", *Mechanism and Machine Theory*, vol. 44, pp. 57–65.
- viii. D. E. Goldberg, "GA in Search, Optimization, and Machine Learning", Person Education Pte Ltd. Kalyanmoy Deb, "Optimization for Engineering Design Algorithms and Examples", Prentice-Hall of India Pvt Ltd.
- ix. Dan B. Marghitu, "Mechanisms and Robots Analysis with MATLAB<sup>®</sup>", Springer.