

# The Influence of Radiative Heat Transfer and Hall Current on MHD Flow in a Vertical Rotating Channel with Slip Condition

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**Abstract:** *In this paper the effects of Hall current on MHD free convection flow in a vertical rotating channel filled with porous medium have been studied. A uniform magnetic field is applied in the direction normal to the plates. The entire system rotates about an axis normal to the planes of the plates with uniform angular velocity  $\Omega'$ . The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce radiative heat transfer. The effects of various parameters on the velocity and temperature field are shown graphically. Also the results on Skin Friction and Nusselt Number are shown in tables.*

**Key Words:** Hall current, MHD, Free convection, Heat Transfer and Porous medium etc.

## 1. INTRODUCTION

The study of flow in rotating porous channel is motivated by its practical applications in geophysics and engineering. Among the applications of rotating flow in a porous media to engineering disciplines, one can find the food processing industry, chemical processing industry, centrifugation filtration processes and rotating machinery. Also the hydrodynamic rotating flow of electrically conducting viscous incompressible fluids has gained considerable attention because of its numerous applications in physics and engineering. In geophysics, it applies to measure and study the position and velocities with respect to fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics now days has become an important branch of fluid dynamics due to the increasing interest to study environment. In Astrophysics; it is applied to study the stellar and solar structure, interplanetary and interstellar matter, solar storms etc. In engineering, it finds its application in MHD generator ion propulsion, MHD bearing, MHD pumps, MHD boundary layer control of re-entry vehicles etc. Several scholars viz. Crammer and Pai [i], Ferraro and Plumpton [ii] and Shercliff [iii] have studied such flows because of their varied importance and applications. The process of heat transfer is encountered in cooling of nuclear reactors, providing heat sink in turbine blades and aeronautics. There are numerous important engineering and geophysical applications of channel flows through porous medium, for example in the

fields of agriculture engineering for channel irrigation and to study the underground water resources, in petroleum technology to study environment of natural gas, oil and water through the oil channels/ reservoirs. The transient natural convection between to vertical walls with porous material having variable porosity has been studied by Paul et al [iv]. In the recent years, the effect of transversely applied magnetic field on the flow of the electrically conducting viscous fluid have been studied extensively owing to their astrophysics, geophysical and engineering application Attia and Kotb [v] have studied MHD flow between the two parallel plates with heat transfer. When the strength of magnetic field is strong, one cannot neglect the effect of Hall current. The rotating flow of an electrically conducting fluid in the presence of magnetic field is encountered in geophysical and comical fluid dynamics. It is important in solar physics involved in sun spot development. Hall effect on unsteady MHD free and forced convection flow in a porous rotating channel has been investigated by several researchers Sivaprasad et al [vi], Singh and Kumar [vii], Singh and Pathak [viii], and Ghosh et al [ix]. Radiative convective flows have gained attention of many researchers in recent years. Radiation plays a vital role in many engineering, environment and industrial process for example heating and cooling chamber, fossil fuel combustion energy processes astrophysical flows and space vehicle re-entry. Raptis [x] studied the radiation free convective flow through a porous medium. Alagoa et al [xi] has analysed the effect of radiation on MHD flow through the porous medium between infinite parallel plates in the presence of time dependent suction. Singh and Kumar [xii] have studied the radiation effect on the exact solution of free convective oscillatory flow through porous medium in a rotating porous channel. The behaviour of the fluid under extreme confinement is of great interest from both the scientific and technological point of view. One of the great complexities is to discover what type of boundary condition is appropriate for solving the continuum fluid problems. Despite of the wide spread acceptance of no slip assumption, there has been existed for the many years, indirect experimental evidence based on the anomalous flow in capillaries and other systems that in some cases, simple liquid can slip against the solid when walls are sufficiently smooth and the no slip boundary condition is no more valid. The no slip boundary condition is only valid when particle close to the surface do not move along with the flow i.e. when adhesion is stronger than cohesion. However this is only true microscopically. Few other limitations of no slip conditions

are; they fail for large contact angle, does not hold at very low pressure, does not work for polyethylene, rubber compounds and suspensions, fail in hydrodynamics for hydrophobic surfaces (Vinogradova [xiii], Zhu and Granick [xiv]).

Recently, the slip boundary condition has significant application in lubrication, extrusion, medical sciences, especially in polishing article heart valves, flows through porous media, micro and nano-fluids, friction studies and biological fluids, Pit et al [xv]. On the other hand chemical reactions have numerous applications such as manufacturing of ceramic, food processing and polymer production. Muthucumaraswamy [xvi] has analyzed that the rate of diffusion is affected by chemical reaction. Many researchers have shown interest propulsion engines for aircraft technology Murti et al [xvii].

Motivated by the above researches our purpose to investigate the Hall effects on an unsteady MHD oscillatory convective heat transfer flow of a radiating and chemically reacting fluid through a porous medium in a rotating vertical porous channel with slip condition.

## 2. MATHEMATICAL FORMULATION

Consider the flow of a viscous, incompressible and electrically conducting fluid through a porous medium bounded by two infinite vertical insulated plates at distance  $d$  apart. We introduce a Cartesian co-ordinate system with  $x'$ -axis oriented vertically upward along the centre line of this channel and  $y'$ -axis taken perpendicular to the planes of the plates which is the axis of the rotation and the entire system comprising of the channel and the fluid are rotating as a solid body about this axis with constant angular velocity  $\Omega'$ . A constant injection velocity  $U$  is applied at the plate  $y' = 0$  and the same constant suction velocity  $U$  is applied at the plate  $y' = a$ . A uniform magnetic field with magnetic flux density vector  $B_0$  is applied perpendicular to the plane of plates. Since the plates of the channel occupying the planes  $y' = 0$  and are of infinite extent, all the physical quantities depend upon only on  $y'$  and  $t'$  only. Under the Boussinesq approximation, the flow of the fluid through the porous medium in a rotating channel is governed by the following equation:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho(1+m^2)} (mv' - u') \quad (1)$$

$$+ g\beta(T - T_0) + 2\Omega'v' - \frac{\nu}{K'}u'$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho} \frac{\partial p'}{\partial z'} + \nu \frac{\partial^2 v'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (mu' + v') - \frac{\nu}{K'}v' - 2\Omega'u' \quad (2)$$

and

$$\frac{\partial T}{\partial t'} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y'} + \frac{Q}{\rho C_p} (T - T_0) \quad (3)$$

Subject to the boundary condition

$$\left. \begin{aligned} u' &= a\gamma \frac{\partial u'}{\partial y'}, v' = a\gamma \frac{\partial v'}{\partial y'}, T = T_0 \quad \text{on } y' = 0 \\ u' &= v' = 0, T = T + (T_w - T_0) \cos \omega t' \quad \text{on } y' = a \end{aligned} \right\} \quad (4)$$

$$\frac{\partial q}{\partial y'} = 4\alpha^2 (T - T_0) \quad (5)$$

Non-dimensional quantities are as follows

$$\begin{aligned} R_e &= \frac{Ua}{\nu}, & x &= \frac{x'}{a}, y = \frac{y'}{a}, z = \frac{z'}{a}, t = \frac{t'U}{a}, \\ u &= \frac{u'}{U}, v = \frac{v'}{U}, & \theta &= \frac{T - T_0}{T_w - T_0}, & M^2 &= \frac{a^2 \sigma B_0^2}{\rho \nu}, \\ N^2 &= \frac{4a^2 \alpha^2}{\kappa}, & P_e &= \frac{aU \rho C_p}{\kappa}, & b^2 &= \frac{Qa^2}{\kappa}, & P &= \frac{aP'}{\rho \nu U}, \\ G_r &= \frac{a^2 g \beta (T_w - T_0)}{\nu U}, & K &= \frac{K'}{a}, & \Omega &= \frac{\Omega' a^2}{\nu} \end{aligned} \quad (6)$$

Substituting equations (5) and (6) in equations (1) to (4), we get following non-dimensional equations

$$\begin{aligned} R_e \frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{M^2}{(1+m^2)} (mv - u) \\ &+ G_r \theta - \frac{1}{K} u + 2\Omega v \end{aligned} \quad (7)$$

$$R_e \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\partial^2 v}{\partial y^2} - \frac{1}{K} v - 2\Omega u - \frac{M^2}{(1+m^2)} (mu + v) \quad (8)$$

$$R_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (N^2 + b^2) \theta \quad (9)$$

Corresponding boundary conditions are as follows

$$\left. \begin{aligned} u &= \gamma \frac{\partial u}{\partial y}, v = \gamma \frac{\partial v}{\partial y}, \theta = 0 \quad \text{on } y = 0 \\ u &= v = 0, \theta = \cos \omega t \quad \text{on } y = 1 \end{aligned} \right\} \quad (10)$$

For the oscillatory internal flow, we shall assume that the fluid flows only under the influence of non-dimensional pressure gradient oscillating in the direction of x-axis only which is the form

$$-\frac{\partial p}{\partial x} = \lambda \cos \omega t$$

In order to combine equations (7) and (8) into single equation, we introduce a complex function  $F = u + iv$

$$\frac{\partial^2 F}{\partial y^2} - R_e \frac{\partial F}{\partial t} - \left( 2\Omega i + \frac{1}{K} + \frac{M^2}{(1+m^2)}(1+im) \right) F = -\lambda \cos \omega t - G_r \theta \quad (11)$$

And boundary conditions in complex form can be written as

$$\left. \begin{aligned} F &= \gamma \frac{\partial F}{\partial y}, \theta = 0 \quad \text{on } y=0 \\ F &= 0, \theta = e^{i\omega t} \quad \text{on } y=1 \end{aligned} \right\} \quad (12)$$

### 3. SOLUTION OF THE PROBLEM

In order to solve under the boundary condition we assume in complex form the solution of the problem

$$\left. \begin{aligned} F(y,t) &= F_0(y)e^{i\omega t} \\ \theta(y,t) &= \theta_0(y)e^{i\omega t} \\ \text{and} \\ -\frac{\partial p}{\partial x} &= \lambda e^{i\omega t} \end{aligned} \right\} \quad (13)$$

Substituting these values in equations (11), (9) and (10), we get

$$F_0'' - K_1^2 F_0 = -\lambda - G_r \theta_0 \quad (14)$$

$$\text{Where } K_1^2 = \left( 2\Omega i + \omega R_e i + \frac{1}{K} + \frac{M^2}{(1+m^2)}(1+im) \right)$$

$$\theta_0'' + K_2^2 \theta_0 = 0 \quad (15)$$

Where  $K_2^2 = N^2 + b^2 - \omega P_e i$

Boundary condition becomes

$$\left. \begin{aligned} F_0 &= \gamma F_0', \theta_0 = 0 \quad \text{on } y=0 \\ F_0 &= 0, \theta_0 = 1 \quad \text{on } y=1 \end{aligned} \right\} \quad (16)$$

Solution of equations (14) and (15)

$$\theta_0(y) = \frac{\sin(K_2 y)}{\sin(K_2)} \quad (17)$$

$$F_0(y) = C_1 \cosh(K_1 y) + C_2 \sinh(K_1 y) + \frac{\lambda}{K_1^2} + \frac{G_r}{(K_1^2 + K_2^2)} \left( \frac{\sin(K_2 y)}{\sin(K_2 y)} \right) \quad (18)$$

Where

$$C_1 = -\frac{\lambda}{K_1^2} + \gamma \left( K_1 C_2 + \frac{K_2 G_r}{(K_1^2 + K_2^2) \sin(K_2)} \right) \text{ and}$$

$$C_2 = \frac{1}{\sinh(K_1) + \gamma K_1 \cosh(K_1)} \left\{ \frac{\lambda}{K_1^2} (\cosh(K_1) - 1) - \frac{G_r}{(K_1^2 + K_2^2)} \left( 1 + \frac{\gamma K_2 \cosh(K_1)}{\sin(K_2)} \right) \right\}$$

### Skin-friction

The expression for the shear stress is given by

$$\tau = -\left( \frac{\partial F}{\partial y} \right)_{y=0}$$

$$\tau = -\left( K_1 C_2 + \frac{K_2 G_r}{(K_1^2 + K_2^2) \sin(K_2)} \right) e^{i\omega t} \quad (19)$$

### Nusselt number

The expression for the rate of heat transfer is given by

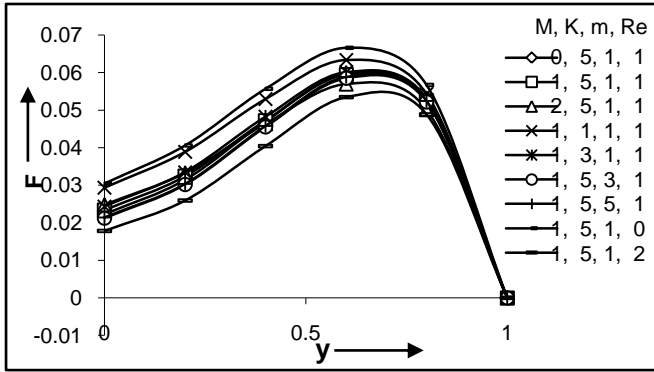
$$Nu = -\left( \frac{\partial \theta}{\partial y} \right)_{y=0}$$

$$Nu = -\frac{K_2 e^{i\omega t}}{\sin(K_2)} \quad (20)$$

### 4. RESULTS AND DISCUSSION

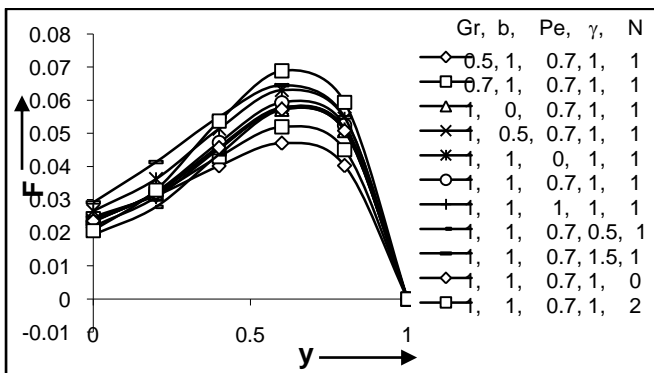
In order to study the effect of different parameters appearing in the flow problem, we have carried out numerical calculations for the Velocity field, Skin friction, Temperature field and Nusselt number.

Figure 1, shows the variation of velocity profiles under the influence of the magnetic parameter, porosity parameter, Hall parameter and Reynold number. It is evident from figure 1 that the velocity decreases with the increase of magnetic parameter M. This is because of the reason that effect of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. It is also observed that in the rotating channel the velocity decreases with increasing K. it is expected physically also because the resistance posed by the porous medium to the decelerated flow due to rotation reduces with increasing permeability K which leads to decrease in the velocity. It is also examined that the velocity increases with the increase of Hall parameter m, where as an increase in Reynold number Re results a decrease in velocity profile.

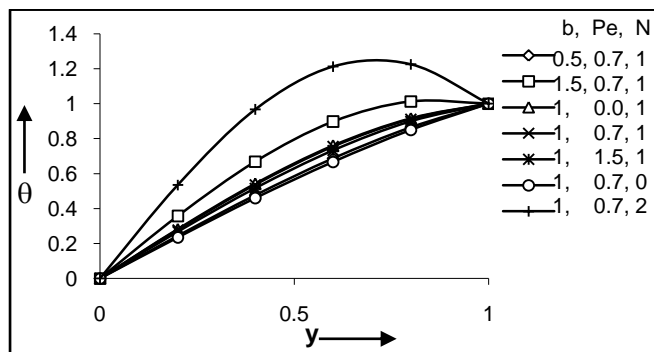


**Fig-1:** Effects of M, K, m and Re on velocity profile

The variation of velocity profile with the Grashof number Gr, heat source parameter b, the Peclet number Pe, slip parameter  $\gamma$  and radiation parameter N are shown in figure 2. The magnitude of velocity leads to an increase with an increase in Gr. It is due to the fact an increase in the value of the thermal Grashof number has the tendency to increase the thermal buoyancy effect. It is also clear from the graph 2 that an increase in Peclet number results a decrease in velocity profile, which shows that the Peclet number causes to weaken the fluid slip at the wall. It is also concluded that an increase in slip parameter at the wall causes the velocity increases at the wall.

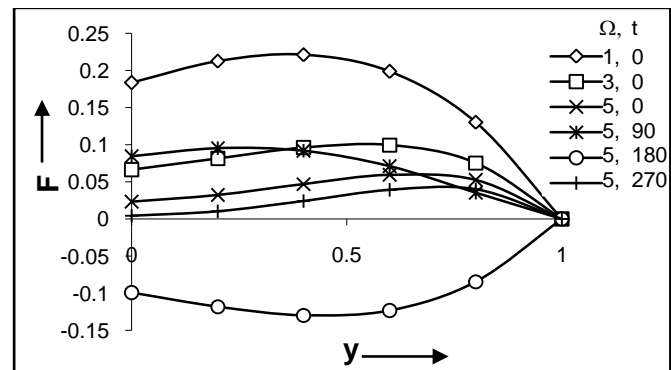


**Fig-2:** Effects of Gr, b, Pe,  $\gamma$  and N on velocity profile



**Fig-3:** Effects of b, Pe and N on temperature profile

Figure 3, shows that the effect of the heat source parameter, Peclet number Pe and radiation parameter N on temperature profiles against y. It is concluded from figure 3 that the temperature decreases with the increase of Peclet number. It is interesting to note that the flow of heat is reversed which the increase of the Peclet number. It is clear from figure 2 & 3 that both velocity and temperature profile increases as increasing b respectively, which shows that the heat source parameter causes to strengthen the fluid slip at the wall. This result quantitatively agrees with expectation since the effect of internal heat generation is to increase the rate of heat transport to the fluid, thereby increasing the temperature of gas and also increasing its velocity. It is clear from figure 3 that temperature profile increases as increasing N, which shows that the radiation parameter causes to strengthen the fluid slip at the wall. In figure 2, the velocity increases as increasing N which shows that the radiation parameter causes to strengthen the fluid slip at the wall. The effect of radiation is to increase the rate of energy transport to the gas, thereby making the thermal boundary layer become thicker and fluid become warmer. This enhance the effect of thermal density variation which are coupled to the temperature and therefore increasing the fluid velocity.



**Fig-4:** Effects of  $\Omega$  and t on velocity profile

Figure 4, shows the variation of velocity profiles under the influence of the rotation parameter the velocity decreases when rotation parameter is increased. This figure also shows that the velocity oscillates in equal intervals of the time and the effect of the slip condition can clearly be seen at the wall. It is worth mentioning here that the strength of slip at the wall in affected by the different parameters involved in the equations.

**Table -1:** Effects of different parameters on Nusselt number

Nu	b	Pe	N
-1.22889	0.5	0.7	1
-1.41267	1	0.7	1
-1.82012	1.5	0.7	1
-1.43173	1	0	1
-1.34669	1	1.5	1
-1.1752	1	0.7	0
-2.76092	1	0.7	2

In table 1, it is seen that an increase in Pe leads to an increase in Nusselt number. This shows that when viscosity of a fluid dominates over conductivity then the rate of heat transfer increases significantly, whereas, Nusselt number decreases with increasing N and b.

**Table -2:** Effects of different parameters on Skin-friction

$\tau$	M	b	N	K	Re	Pe	Gr	m	$\gamma$
-0.02223	0	1	1	5	1	0.7	1	1	1
-0.02334	1	1	1	5	1	0.7	1	1	1
-0.02476	2	1	1	5	1	0.7	1	1	1
-0.0236	1	0	1	5	1	0.7	1	1	1
-0.02355	1	0.5	1	5	1	0.7	1	1	1
-0.0236	1	1	0	5	1	0.7	1	1	1
-0.02071	1	1	2	5	1	0.7	1	1	1
-0.02926	1	1	1	1	1	0.7	1	1	1
-0.02439	1	1	1	3	1	0.7	1	1	1
-0.03038	1	1	1	5	0	0.7	1	1	1
-0.01782	1	1	1	5	2	0.7	1	1	1
-0.02659	1	1	1	5	1	0	1	1	1
-0.01972	1	1	1	5	1	1.5	1	1	1
-0.0248	1	1	1	5	1	0.7	0.5	1	1
-0.02421	1	1	1	5	1	0.7	0.7	1	1
-0.06626	1	1	1	5	1	0.7	1	3	1
-0.02134	1	1	1	5	1	0.7	1	5	1
-0.33063	1	1	1	5	1	0.7	1	1	0
-0.05831	1	1	1	5	1	0.7	1	1	0.5

From table 2, we present the values of skin friction for various values of different parameters as shown in table 2. It is noted that skin friction increases with increasing different parameters such as b, N, K, Re, Pe, Gr, m and  $\gamma$ , whereas, it decreases with increasing magnetic parameter M.

### 5. CONCLUSIONS

The effects of Hall current on MHD free convection flow in a vertical rotating channel filled with porous medium have been studied. A uniform magnetic field is applied in the direction normal to the plates. The dimensionless equations are solved using perturbation technique. From results we concluded that

1. Velocity decreases with the increase of magnetic parameter M, K Pe and Re, whereas increases with increasing of m, b, Gr and  $\gamma$ .
2. The temperature decreases with the increase of Pe, whereas, increases with the increase of b and N.
3. Viscosity of a fluid dominates over conductivity then the rate of heat transfer increases significantly,

whereas, Nusselt number decreases with increasing N and b.

4. Skin friction increases with increasing different parameters such as b, N, K, Re, Pe, Gr, m and  $\gamma$ , whereas, it decreases with increasing magnetic parameter M.

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