

Evaluation and Development of Re-aeration Equation Using Multivariate Linear Regression

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Abstract : *To model and to allocate the waste load in a stream, it is necessary to estimate the re-aeration coefficient (k_2). Re-aeration rate coefficient as compared to loads, sources and sinks, cannot be measured under natural conditions. Indirect measurement with the calculation provide the efficient methodology for the estimation of re-aeration rate coefficient. Natural aeration system of river depending upon its assimilation capacity decomposes the organic matter which is mainly controlled by the flow rate and wastewater load. Currently, available k_2 when applied to different stream conditions as compared to their developed conditions produces poor estimate, because they were derived from small databases composed of potentially highly inaccurate measurements. In the present study, the re-aeration model for the different reaches of Yamuna river have been developed. Most commonly used predictive equation developed earlier have been tested and results obtained from these equation are compared with the field observation. A new predictive equation have been developed using Multivariate Linear Regression (MLR) with different flow conditions. The results of these studies are found with 95% level of significance with observed values and indicate that the modified re-aeration equation is a useful tool for measuring the atmospheric re-aeration rate coefficient.*

Keywords: Environmental Flow, Assimilation Capacity, Re-aeration, Multivariate Regression

Introduction

Presence of organic compound in surface water due to different anthropogenic activities is the major cause of degradation of aquatic life, water quality and human health. Abstraction of fresh water for supply, low flows in river at longer stretches and discharge of drains containing untreated or partially treated wastewater also substantially degrade the water quality of fresh water streams. Parameters mainly reflect the water quality are the level of dissolved oxygen; the presence of nitrates, chlorides, phosphates; the level of suspended solids; chemical oxygen demand, heavy metals, and the presence of bacteria (Swaroop et. al., 1985). Seasonal variations in precipitation, surface run-off, interflow, groundwater flow and pump in and out flows have a strong effect on river discharge and subsequently on the concentration of pollutants in river water. Along with the physio-chemical parameters hydraulic properties, interaction of pollutants with metals or other pollutants and interaction of surface water with ground water also plays a significant role in alteration of water quality of rivers. Despite the huge numeric datasets collected nowadays, it is well known that the assessment of water quality still relies heavily upon subjective judgments and interpretation.

Waste-load allocation is the process by which allowable concentrations of constituents in discharge from wastewater-treatment plants (WWTPs) are determined such that acceptable water quality can be maintained in the stream. The allowable discharges from WWTPs typically are determined by simulation of water quality processes in streams utilizing a computer model such as QUAL2E (Brown and Barnwell, 1987).

Disposal of municipal and industrial wastewater in rivers started in 1963 with the fact that municipal and industrial wastes are discharged into a stream does not necessarily mean that the stream is polluted, because the stream does have the capacity for self-purification (Bennett, 1972). Assessment of water quality using mathematical models started in 1920s. In 1925 Streeter and Phelps predicted the equation for the re-aeration coefficient and balance of oxygen concentration in rivers. Subsequently model has been amended in several ways. The oxygen source considered by Streeter and Phelps (1925) was re-aeration, or the physical absorption of oxygen from the atmosphere by the flowing stream (Streeter et. al., 1925).

In the first study one source were considered with one sink. The oxygen sink was the biochemical oxidation of carbonaceous wastes by bacteria in the stream (James et. al., 1987). It is generally assumed that biochemical oxidation is a first-order process; that is, the rate of biochemical oxidation is proportional to the amount of carbonaceous material remaining to be oxidized (Pimpunchat et. al., 2009). This process may be described by

$$\frac{dL}{dt} = -K_1 L \quad \dots 1$$

where L is the BOD (biochemical oxygen demand) of the carbonaceous material, t is time, and K_1 is the de-oxygenation coefficient or the rate constant for the oxidation of carbonaceous wastes. Adeney and Becker (1919, 1920) (James et. al., 1987) showed that the absorption of oxygen by water is a first-order process; that is, the rate of absorption is directly proportional to the saturation deficit (Pimpunchat et. al., 2009). This process is described by

$$\frac{dD}{dt} = K_2 D \quad \dots 2$$

where K_2 is the re-aeration coefficient or the rate constant for the absorption of oxygen from the atmosphere and D is the DO deficit. DO balance obtained from this process was

$$\frac{dD}{dt} = K_2 D - K_1 L \quad \dots 3$$

Other authors, notably Dobbins (1964) and Camp (1963), have expanded the fundamental Streeter-Phelps equation to include in the oxygen balance these and other sources and sinks of DO (Dobbins, 1956, 1965).

Researchers have conducted their studies for water-quality management on the basis of simulation of constituent concentrations throughout the entire year (e.g., Demuyne et al. 1997). Thus, equation generated from the simulation studies, were related to the stream flow conditions. On the basis to relate the re-aeration coefficient and hydraulic characteristics of the stream, more than 20 equations are available in the literature. These equations and the data used to derive them are summarized in Flores (1998). Various equations determined on re-aeration were compared in the literature and found that most of the K_2 -estimation equations in the literature were derived from relatively small sets of laboratory or field data for a relatively localized group of streams and there is always uncertainty lies in the reflection of the laboratory test with the field.

Using all the theories, study was conducted to identify the best fit equation on the critical situation of Delhi stretch of river Yamuna, India. The purpose of this paper is to present a detailed description of the development and testing of a new set of equations for estimating K_2 with the brief review of the current state of practice for determination of K_2 and the problems associated with the previously available equations for estimating K_2 .

I. Study Area

River Yamuna is influenced by the problems imparted by industrialization, urbanization and rapid agricultural developments similar to other riverine system. Most threatened part of the river starts at Delhi when it enters through Wazirabad barrage and leaves Okhla Barrage followed by the stretch up to Agra. At Wazirabad the river is trapped through a barrage and around 950 MLD of water abstracted for drinking water supply to urban agglomeration at Delhi (CPCB, 2006). From Wazirabad barrage, barely any water is allowed to flow down particularly during summer, as the available water in the river is not adequate to fulfil the water supply demand of Delhi. River water again diverted at Okhla barrage after receiving waste water from 22 drains in Delhi, where 18 are the major drains falls directly onto river and 4 drains falls into Agra and Gurgaon canal (CPCB, 2006). Similarly as to downstream of Wazirabad, at downstream Okhla barrage, the water flows in the river is the drain water of domestic & industrial origin contributed mainly by Shahdara drain.

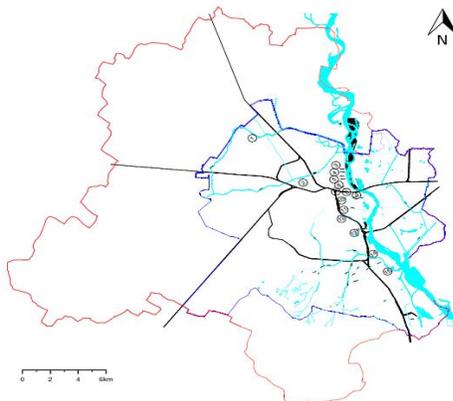


Figure 1: Drains joining river Yamuna in Delhi, India

Study were conducted out and validated with the river Yamuna water quality in the stretch between the Wazirabad Barrage and up to Okhla Barrage (APHA, 1992). Out of all the wastewater drains joins the river Yamuna, Najafgarh drain carries the maximum discharge. Accordingly on the basis of impact on water quality, sampling were carried out at four points in river Yamuna at downstream of Wazirabad barrage at 0.5 km after receiving the wastewater load from Najafgarh drain, near ITO barrage, Nizamuddin bridge after joining drain no. 12 & 14 and at upstream of Okhla barrage.

II. Methodology

The equations developed for estimation of K_2 were obtained either by multiple linear regression of the logarithms of the available data or derivation of an equation form from a theoretical concept of the re-aeration process. Some of the coefficients and exponents of the theoretical equations were determined from physical reasoning, whereas others were determined through a least-squares fitting of available data. Thus, the equations determined from multiple linear regressions are categorized as empirical models, and the equations determined by least-squares fitting of a theoretically derived equation are categorized as semi-empirical models (Rathbun et al., 1977).

Various studies are available in literature for the determination of re-aeration coefficient. The best field study of the re-aeration coefficient has been done by Churchill, Elmore, and Buckingham (1962) (Bennett, et. al., 1972). Their measurements were made in stretches below dams, where the water released was deficient in DO because of prolonged storage under conditions of thermal stratification, and where generally the BOD was negligible. They performed many multiple-regression analyses on the pertinent stream flow, gas, and liquid parameters corresponding to the k_2 measurements. None of the prediction equations was found to be statistically any more significant than any of the others, so that the simplest one was suggested by these authors for general usage. This equation, which has a coefficient of multiple correlation of 0.82, is

$$\dots 4$$

where k_2 is in reciprocal days, U is in feet per second, H is in feet, and T is in degrees Celsius. Owens, Edwards, and Gibbs (1964) reported re-aeration coefficient measurements for several streams (Owens et.al., 1968). These coefficients were determined using the disturbed-equilibrium method of de-aeration with sodium sulfite and cobalt catalyst. Multiple-regression analysis of the data by Owens, Edwards, and Gibbs (1964) (Bennett, et. al., 1972) yielded

$$\dots 5$$

Owens, Edwards, and Gibbs (1964) (Owens et.al., 1968) included the data of Gameson, Truesdale, and Downing (1955) and that of Churchill, Elmore, and Buckingham (1962) in the analysis, the resultant equation was:

$$\dots 6$$

For the study, stretch of river Yamuna have been selected as it contains largely biological wastes which undergo various biochemical and biodegradation processes using dissolved oxygen. River is divided in to 16 reaches with variable distance, x . At $x < 0$, near the source it is assumed that there is no pollution source and downstream $0 < x < L$ where pollution is added at a rate 'q' (Pimpunchat et. al., 2009). Flow diagram of river Yamuna indicating the point of joining the drains to the river along the study area and sampling points in the stream is shown in Figure 3. The addition of pollutant, which is strictly a function of time and position, will be taken to be constant along the downstream portion of the river.

Different re-aeration equations are analysed and applied to the study area using the Dobbins DO balance equation. Dobbins (1964) expanded the fundamental Streeter-Phelps equation to include the oxygen balance with the other sources and sinks of DO. O'Connor and Dobbins have also re-analysed the Streeter Phelps data to yield the prediction equation (Dobbins, 1965). A DO balance equation of Dobbins written as follows:

$$\frac{dD}{dx} + \frac{D}{L} - \frac{D_0}{L} - \frac{K_3}{U} = \dots 7$$

where D is the DO concentration, D_0 is the deficit DO, L is longitudinal distance along the reach, U is the mean stream velocity, K_3 is the settling rate coefficient and S_B is the sediment oxygen demand into the benthal layer and the effect of photosynthesis and respiration by plants.

III. Result & Discussion

Water quality parameters were tested out through field sampling (APHA, 1972). Sample collected at the 0.5 km downstream of Wazirabad barrage after mixing of Najafgarh Drain and found to be completely reflecting the drain characteristics. Values of BOD, COD and DO are way away from the respective permissible limits. Dissolved Oxygen (DO) values found low or nil.

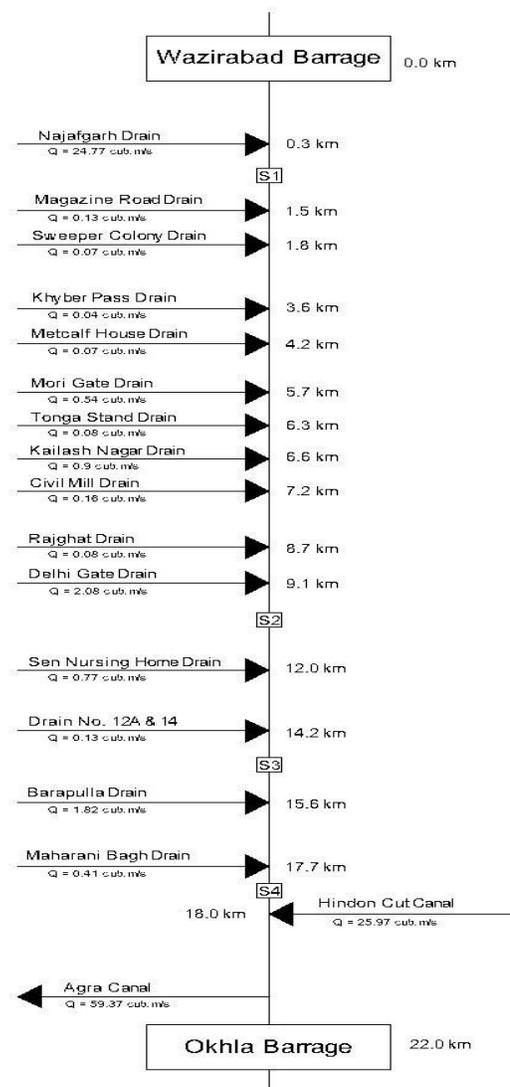


Figure 2: Flow diagram of river Yamuna

This reflects the presence of organic pollution load and prevalence of eutrophic. The anoxic condition in the river is frequently observed and as evident from the presence of masses of rising sludge from the bottom, gas bubbles and floating solids on the surface. High values of BOD are due to the presence of organic contamination, solid waste leaching and disposal of waste water without any treatment in river water. Samples were also collected at ITO Barrage (9.3 km D/s of Wazirabad Barrage), Nizamuddin Bridge (14.4 km D/s of Wazirabad Barrage) and U/s of Okhla Barrage (Before discharging to Agra Canal). At all the points DO were found to be very low leads to the formation of anoxic environment in the river. Subsequently, all the drains joining the river Yamuna in the study area were also analyzed and water quality were tested out as per APHA-AWWA standard method of testing the water and wastewater parameters. Variation in the flow of the river caused by the addition of point loads were also measured and shown in Figure 3.

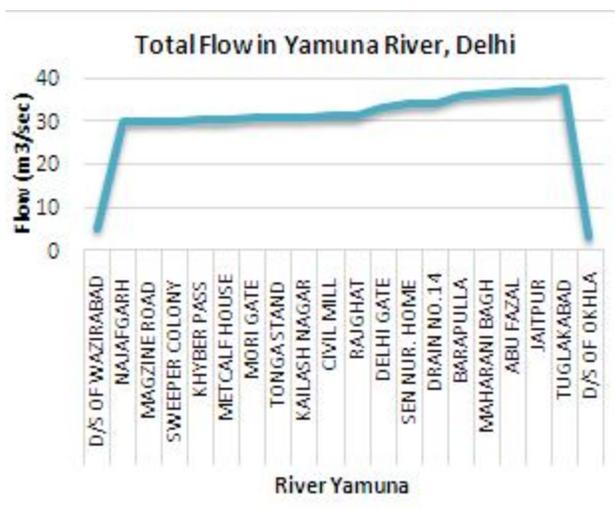


Figure 3: Flow variation in river Yamuna in study area

All the methods for determining the re-aeration coefficient in rivers uses integrated form of DO balance equation. Using the same method, re-aeration coefficient derived in equation no. 5, 6 and 7. All these equations applied to the study area and best fit equation have been identified. Out of the three equations, equation no. 5 and 7 provided by Churchill (1962) and Owens (1964) were found to be close to the field data. Both the equation follows the same pattern of the variation of DO in river, but Owens equation best fit to the field data as shown in Figure 5. It has been identified that the assumptions used in the calculation of Owen's re-aeration constant correlated with the study area.

Whereas, significant gap have been observed between DO concentration calculated by using Owen's re-aeration coefficient and measured value, either caused by the re-aeration constant or by the parameter that are not included in the estimation and depletion of dissolved oxygen from the stream. The major sources of error are differences between the dynamic stream condition and the quiescent condition in the various bottle techniques developed for measuring the DO balance parameters, the difficulty and expense of obtaining a sufficient number of representative samples to describe accurately the stream conditions at the cross section of interest, deviations from the assumptions inherent in the derivation of the DO balance equation, and the accuracy of the DO measurement procedure.

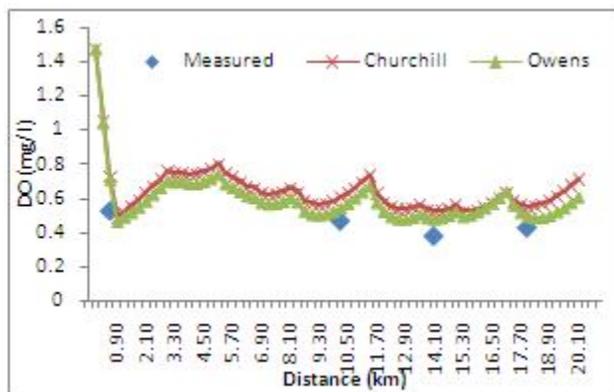
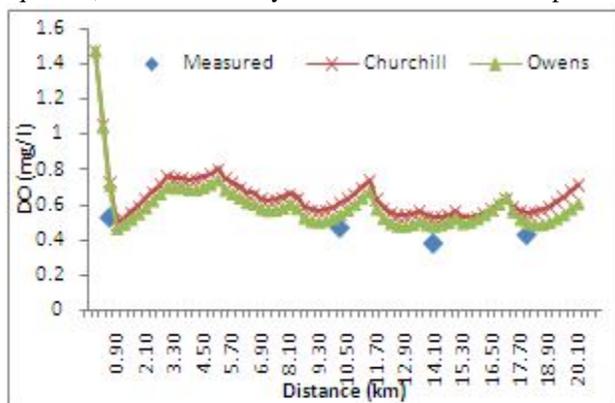


Figure 4: DO calculated by using Owen's, Churchill's re-aeration coefficient and measured.

In order to reduce the error in the analysis, Owen's re-aeration equation is modified by adding the ' ' quantity to the equation which becomes:

$$\dots 8$$

Value of the ' ' is added and equation is modified to:

$$\dots 9$$

Modified equation were applied to the model and compared with the field results and results obtained are shown in figure 6. To verify the usefulness of the equation developed on the basis of data for the study area of year 2009 and to predict the efficiency of the model. The system constants were kept identical to those values determined from the model calibration. Both the model results and field measurement were compared in figure 6 and 7. Model is calibrated for year 2009 field data of river and model is validated with the river field data of 2010 and 2011.

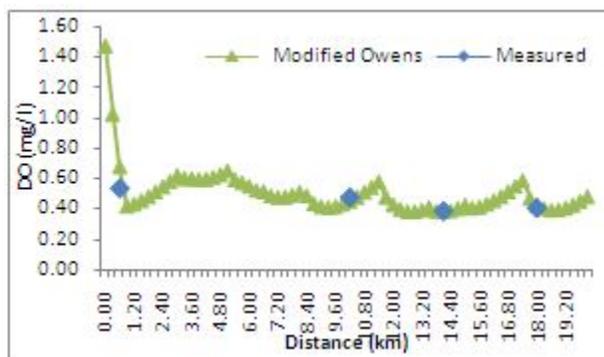


Figure 5: Relation between measured DO (mg/l) of the study area (2009) and DO (mg/l) using modified Owen's

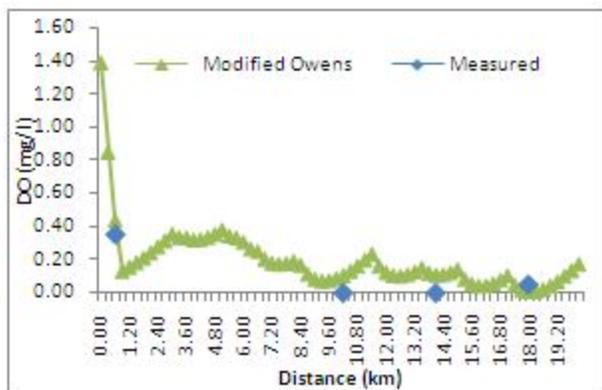


Figure 6: Relation between measured DO (mg/l) (2010) and DO (mg/l) using modified Owen's

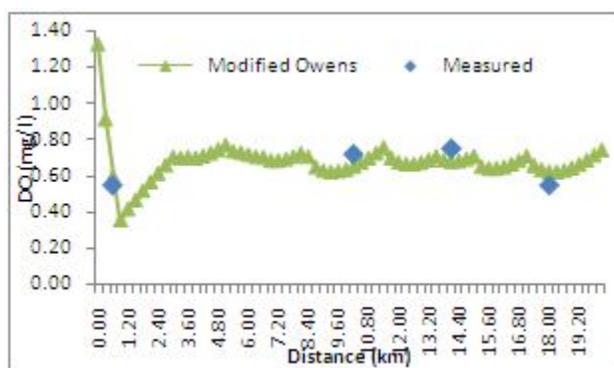


Figure 7: Relation between measured DO (mg/l) (2011) and DO (mg/l) using modified Owen's

A general statistical analysis lead to the various sources of error on the calculation of the re-aeration coefficient and it is probably is not feasible with the present state of our knowledge to completely eliminate the errors occurred in the estimation. However, it is instructive and important to consider the range of possible errors in the measured DO balance parameters and the effect of these errors on the DO balance equation used to calculate the re-aeration coefficient. The figure shows that model represent the field data quite well, the model prediction lines coincide in almost throughout the study area in figures. Modified Owen's re-aeration coefficient suited well to the ambient condition of study area and can be utilized also for the subsequent studies on the same study area or the river basin.

IV. Conclusion

Use of aeration in the degradation of pollutant concentration has been calculated for river pollution comprising a coupled pair of nonlinear equations. Analytic steady- state solutions of the Dobbins's equation have been used. There are several formulae for predicting the variation of re-aeration with flow and most of these are based on some combination of depth and velocity. In

this study, only the Owen's-Dobbins relationship was tested and verified on the study area, and it may be possible to find relationships that apply to various stream types. For the verification of the equation developed from the modification of Owen's re-aeration coefficient equation, data set K_2 measurement applied to the study area for the year 2010 and 2011, which was calibrated with the data set of year 2009. The variation of re-aeration with flow shows a very strong effect on the relationship between minimum dissolved oxygen concentration and flow. Equations obtained may be reliable for estimation of K_2 for waste-load-allocation studies for which in stream measurements of K_2 . However, these equations are not a replacement for field measurement of K_2 on the streams of interest and in stream measurements of K_2 should be done.

References

- i. APHA-AWWA, *Standard method for examination water and waste water (19th edition)*, American Public Health Association, Washington, DC, 1992.
- ii. Bennett J. P. and Rathbun R. E., *Re-aeration in open channel flow*, Geological Survey Professional Paper 737, United States Government Printing Office, Washington, 1972.
- iii. CPCB, *Water Quality Status of Yamuna River*, Central Pollution Control Board, Ministry of Environment and Forest, Delhi, 2006.
- iv. Dobbins, W. E., *The nature of the oxygen transfer coefficient in aeration systems*, in *Biological treatment of sewage and industrial wastes: New York*, Reinhold Book Corp.; p. 141-253, 1956.
- v. Dobbins, W. E., *Closure to BOD and oxygen relationships in streams: Am. Soc. Civil Engineers Jour.*, v. 91, no. SA-5, p. 49-55, 1965.
- vi. James, A., in: M.B. Beck (Ed.), *Mathematical Models in Water Pollution Control, Modelling of Dissolved Oxygen in a Non-tidal Stream*, John Wiley and Sons Ltd, Southerngate, Chichester, pp. 137-163, 1987.
- vii. O'Connor, D. J., and Dobbins, W. E., *Mechanism of re-aeration in natural streams: Am. Soc. Civil Engineers Trans.*, v. 123, p. 641-684, 1958.
- viii. Owens, M., and Knowles, G., *The prediction of the distribution of dissolved oxygen in rivers: Water Research*, v. 2, no. 1, p. 20-21, 1968.
- ix. Pimpunchat B., Sweatman W.L., Triampo W., Wake G.C., Parshotam A., *A mathematical model for pollution in a river and its remediation by aeration*, *Applied Mathematics Letters*, 22, pp. 304-308, 2009.
- x. Streeter H.W., Phelps E.B., *A study of the pollution and natural purification of the Ohio river*, in: *US Public Health Service, Public Health Bulletin*, vol. 146, Feb. U.S. Government Printing office, Washington, D.C., 1925.
- xi. Swaroop B. D., *Water quality variations and control technologies of Yamuna river*, *Journal of Environmental Pollution Series A*, 37(4), 355-363, 1985.
- xii. Brown, L. C., and Barnwell Jr., T. O., "The enhanced stream water quality models QUAL2E and QUAL2E-UNCAS: Documentation and user manual." Rep. No. EPA/600/3-87/007, U.S. Environmental Protection Agency, Athens, Ga, 1987.
- xiii. Melching C. S., Flores H. E., *Re-aeration equations derived from U.S. Geological survey database*, *Journal of Environmental Engineering*, Vol. 125, No. 5, May, 1999.
- xiv. Rathbun, R. E., "Re-aeration coefficients of streams—State of the art." *Journal of Hydr. Div., ASCE*, 103(4), 409-424, 1977.