

Pressure Distribution & Aerodynamic characteristics of NACA Airfoils using Computational Panel Method for 2D Lifting flow

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Abstract: Determination of pressure distribution on airfoil profiles enables to understand the dynamic flow characteristics around the surface of wings or blades. Computational methods are widely applied in the field of aerodynamics to predict the aerodynamic characteristics such as the pressure, lift and drag coefficients on an airfoil. In the present paper, the pressure distribution of NACA airfoil profiles is calculated using the numerical panel method for 2D lifting air flow conditions. The analysis of airfoil geometry subjected to various AOA (angle of attack) including the stalling angles is considered to observe the variation in pressure coefficients along the chord. The zero lift AOA for the profiles are also evaluated in order to assess influence of thickness to chord ratios on airfoil characteristics.

Keywords: Pressure coefficient, 2D Panel, Airfoil, lift and drag, Angle of attack,

1. Introduction

NACA airfoil profiles are widely used in the aircraft industry for producing lift on the wing span. The geometry of airfoil dictates the airfoil performance and determines its relevance for a specific application. The present analysis deals with the analysis of pressure distribution over the airfoil surfaces along with the lift and drag characteristics. They are commonly expressed in the form of drag polar for the NACA series and represent the aerodynamic performance at various flow field conditions. The Reynolds number is often used to characterize the flow conditions and predict the behavior of airfoils at various operating flow conditions both for the viscous and in viscid fluids. Typically the results from the experiments serve as the emulation tool in the aircraft industry for comparison with actual performance of the wing. Panel methods are modern numerical techniques which are quick to execute and predict accurate results compared to the traditional experimental methods that are cumbersome procedures and time consuming.

2. Airfoil geometry

NACA airfoils are designed during the period from 1929 and 1947 by Eastman Jacobs at NASA Langley field laboratory [2, 8]. The airfoil geometry for most of the NACA profiles can be divided into x-coordinates known along the chord line and y – coordinates known as the ordinates, which represent the mean

line or camber of the profiles. The normalized coordinates are obtained by the dividing the x and y values with the chord of airfoil which can be then used for analysis. The profiles of NACA 4412, NACA 16-006, NACA 0024 and NACA 66-018 are shown below. The individual profiles can be differentiated using the distinct parameters as leading edge radius, trailing edge angle. The airfoils have finite thickness at trailing edges which affect the aerodynamic properties in terms of noise and stalling behavior. The leading edge radius and trailing edge angle for NACA MPXX 4 digit airfoils are [1, 3]

$$\frac{r_{LE}}{c} = 1.1019 \cdot \left[\frac{t}{c} \right]^2 \dots (A)$$

$$\delta_{TE} = 2 \tan^{-1} [1.16925 \frac{t}{c}] \dots (B)$$

The thickness and mean line distributions for the 4 digit airfoils are expressed in terms of the t/c ratio, and slope of camber line is obtained by differentiating the camber line function, y_c [6, 1, 7]

$$\theta = \tan^{-1} \frac{dy_c}{dx}$$

$$\frac{y_t}{c} = \frac{t}{c} \left[a_0 \sqrt{\frac{x}{c}} - a_1 \frac{x}{c} - a_2 \left(\frac{x}{c} \right)^2 + a_3 \left(\frac{x}{c} \right)^3 - a_4 \left(\frac{x}{c} \right)^4 \right]$$

$$\left. \begin{aligned} \frac{y_c}{c} &= \frac{M}{(1-P)^2} \left[1 - 2P + 2P \frac{x}{c} - \left(\frac{x}{c} \right)^2 \right] \\ \frac{dy_c}{dx} &= \frac{2M}{(1-P)^2} \left[P - \frac{x}{c} \right] \end{aligned} \right\} \frac{x}{c} \geq P$$

$$\left. \begin{aligned} \frac{y_c}{c} &= \frac{M}{P^2} \left[2P \frac{x}{c} - \left(\frac{x}{c} \right)^2 \right] \\ \frac{dy_c}{dx} &= \frac{2M}{P^2} \left[P - \frac{x}{c} \right] \end{aligned} \right\} \frac{x}{c} < P$$

Where the a_0, a_1, a_2, a_3 & a_4 are the coefficients describe the shape of airfoil and its distribution of ordinates along the chord line.

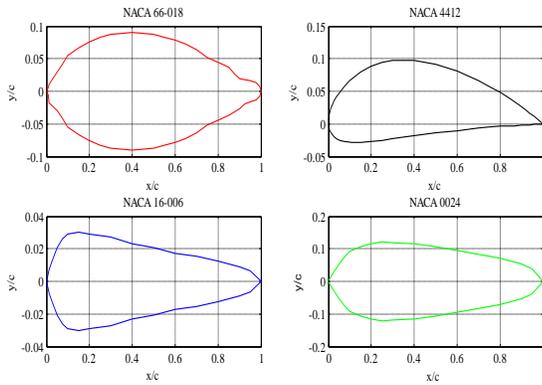


Figure 1 : Geometry of NACA 4 digit, modified & 6 series profiles

3. Computational panel method

Traditional methods for modeling flow around slender bodies of any shape include potential flow which utilizes the superposition of source and sink on x axis and in uniform distributed flow. However, the theory does not predict accurate values for flow whose leading edge has rounded shapes. Basic panel methods were developed by Hess and Smith at Douglas aircraft in late 1950s [2] for aircraft industry. Panel methods model the potential flow by distributing sources over the body surface. A source is point at which the fluid appears in the field at uniform rate while a sink is point which disappears at uniform rate, m^3/s . Each source or sink has specific strength denoted by circulation, Γ . Simple 2D uniform lifting flows [1] can be described using the following equations

$\varphi = by - ax \dots$ stream line function

$\phi = bx + ay \dots$ Velocity potential function

The resultant velocity V , at any point along the flow direction can be written as

$$\sqrt{a^2 + b^2}$$

The point source or sink is distributed uniformly in all directions of the flow field and obey the continuity equation and irrotational motion everywhere except at the point itself. The total velocity potential for numerical panel method can be rewritten as follows [2]

$$\phi = Ux + \mu$$

Where, μ is the perturbation potential away from free stream conditions. The airfoil geometry is discretized into finite number of panels over the surface. The panels are represented by the 2D shape of the surface by series of straight line segment [2, 4]. The following procedure describes the calculation for 2D lifting flows.

- Numbering of end points or nodes of the panels from 1...N
- The center points of each panel are chosen as *collocation points*. The boundary condition of zero flow orthogonal to surface is applied to the points.

- Panels are defined with unit normal and tangential vectors, \hat{n} , \hat{t} .

Velocity vector, denoted by v_{ij} are estimated by considering the two panels, i & j the source on the panel j which induce a velocity on panel i . The perpendicular and tangential velocity components to the surface at the point I , are given by scalar products of $v_{ij} \cdot \hat{n}$ and $v_{ij} \cdot \hat{t}$

- The above quantities represent the source strength on panel j and expressed mathematically as

$$v_{ij} \cdot \hat{n} = \sigma_j N_{ij}$$

$$v_{ij} \cdot \hat{t} = \sigma_j T_{ij}$$

Where N_{ij} and T_{ij} are the perpendicular and tangential velocities induced at the collocation panel i and known as *normal and tangential influence coefficients*. The surfaces represented by the panels are solid and the following conditions are applied for the normal and tangential velocities at each of collocation points consisting of sources strengths, vortices, and oncoming velocity, U .

$$\sum_{j=1}^N \sigma_j N_{ij} + \gamma N_{i,N+1} + \vec{U} \cdot \hat{n}_i = v_{n_i} \dots (1)$$

$$\sum_{j=1}^N \sigma_j T_{ij} + \gamma T_{i,N+1} + \vec{U} \cdot \hat{t}_i = v_{t_i} \dots (2)$$

$$\underbrace{\sum_{j=1}^N \sigma_j N_{ij}}_{\text{Sources}} + \underbrace{\gamma N_{i,N+1}}_{\text{Vortices}} + \underbrace{\vec{U} \cdot \hat{n}_i}_{\text{Oncoming flow}} = 0 \dots (3)$$

Sources Vortices Oncoming flow

The above system of linear algebraic equations are solved for the N unknown source strengths, σ_i , using matrix system and expressed as

$$M \cdot a = b \dots (4)$$

Where N is an $N+1 \times N+1$ matrix containing the N_{ij} and σ_i is column matrix of N elements and A is the column matrix of N elements of unit normal velocity vectors. Matrix inversion procedures available in MATLAB are applied to solve for the source strengths using the above system of equations. The pressure acting at collocation point i is given by the Bernoulli equation as [2, 4, 5]

$$C_{pi} = 1 - \left[\frac{v_{Ti}}{U} \right]^2 \dots (5)$$

Where v_{Ti} the tangential velocity vector is determined using the influence coefficients. The influence coefficients are important for panel method in order to determine the pressure distribution over the surface of the any given airfoil coordinates. The airfoil trailing edge presents a unique condition for the flow field parameters. Using panel method, the following criteria is applied for the stream lines around the airfoil

- The streamlines leave the trailing edge with a direction along the bisector of the trailing edge angle.
- The velocity magnitudes on the upper and lower surfaces near the trailing edge of airfoil approach the same limiting values.
- The trailing edge angle is modeled as the stagnation point for finite value of trailing edge angle hence the source strength must be zero at the trailing edge.

The above assumptions are known as the *Kutta condition* which is essential for the successful evaluation of velocity vectors and pressure for 2D uniform flows. It can be written in the algebraic form of equation as

$$\sum_{j=1}^N \sigma_j T_{t,j} + \gamma T_{t,N+1} + \vec{U} \cdot \hat{t}_i = - \left(\sum_{j=1}^N \sigma_j T_{t,j} + \gamma T_{t,N+1} + \vec{U} \cdot \hat{t}_i \right) \dots (6)$$

It must be noted that the velocity components induced at any point P due to sources on panel centered at point Q can be expressed mathematically as scalar product of vector form

$$v_{PQ} = v_{xQ} \cdot \hat{t}_j + v_{yQ} \cdot \hat{n}_j \dots (7)$$

$$N_{ij} = v_{PQ} \cdot \hat{n}_i \dots (8)$$

$$T_{ij} = v_{PQ} \cdot \hat{t}_i \dots (9)$$

Therefore, the resulting velocity components along with known source strengths and influence coefficients are added for each panel in order to obtain pressure distribution over the airfoil surface. The number of panels used (order) in the simulation ranged from 50 – 70 for airfoils in the MATLAB routine *foil.m*. Depicted below is the flow chart for the computational panel method described in previous section.

Flowchart: Computer routine for panel method

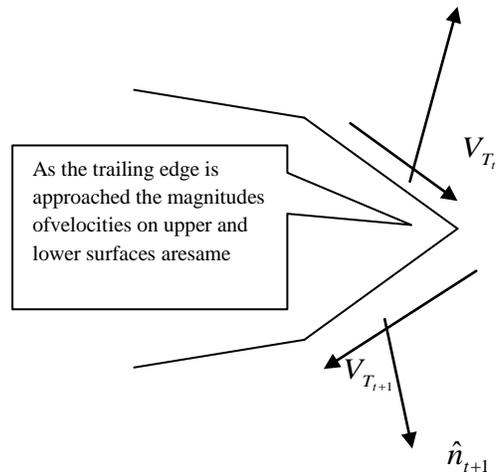
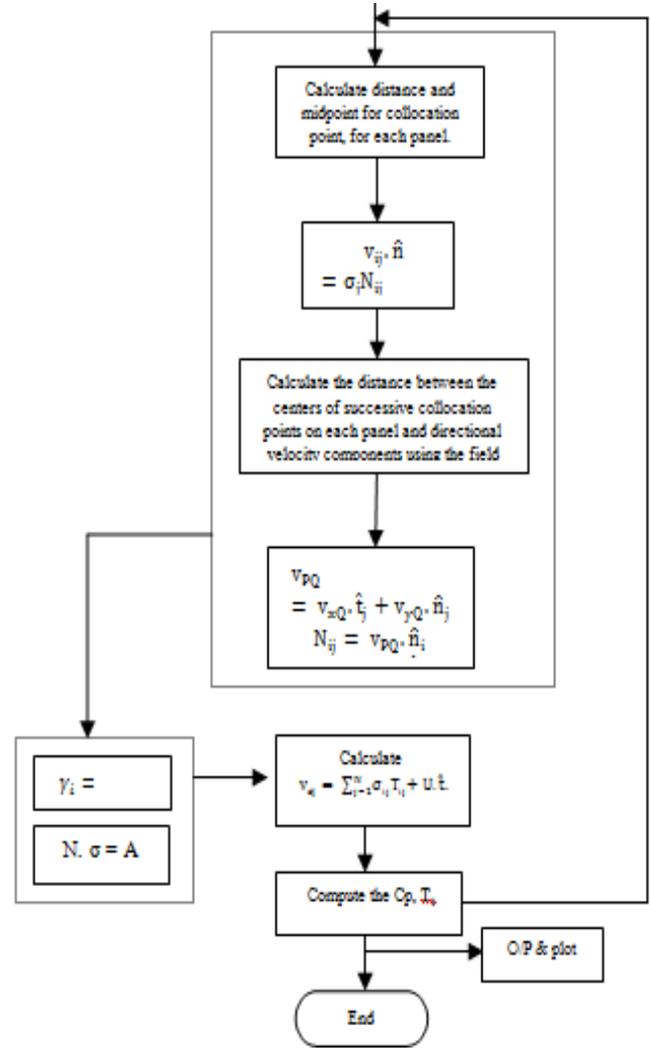
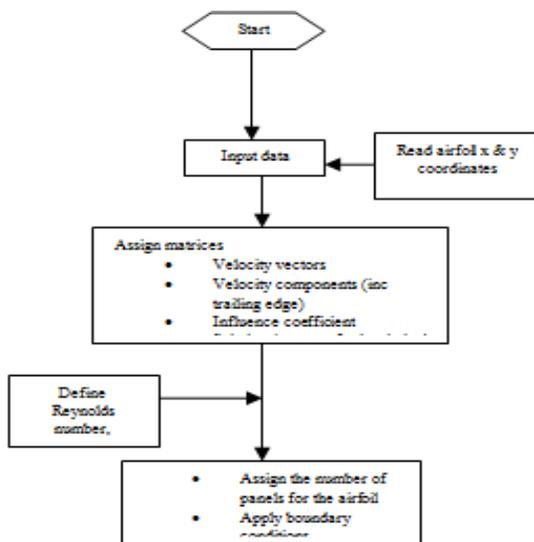


Figure 2. Graphical illustration of kutta condition

4. Results and Discussion

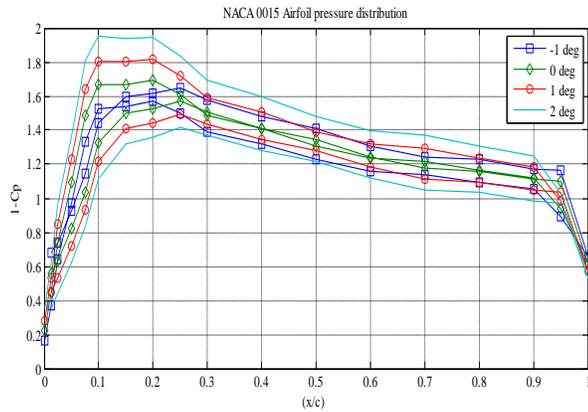


Figure 3: Pressure distribution of NACA 0015 airfoil

The following are the list of NACA airfoils for which the panel method is used to calculate the pressure distribution. The ideal or design lift coefficient is calculated for the zero angle of attack using the formula [1, 3]

$$\alpha_i = \frac{C_{li} h}{2\pi(1 + a)} \dots (9)$$

Where C_{li} is design lift coefficient, h is the function that describes the uniform load along the mean line and the type of mean line used in the airfoil. The thickness and camber is written mathematically as

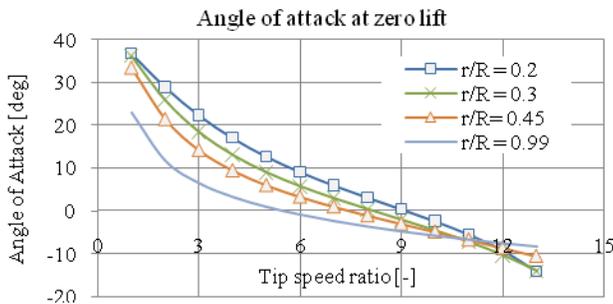


Figure 4: Angle of attack at zero lift along the wing span

$$\frac{\delta}{c} \cdot 100 = \% \text{ camber}; \frac{t_{max}}{c} \cdot 100 = \% \text{ thickness} \dots (10)$$

Fig.2 shows the computed values for pressure distribution of NACA 0015 airfoil. It has 15 % thickness and 0 % camber and identical to the symmetrical airfoil. The 4 digit and 4 digit modified airfoils have same thickness distribution below and above the mean line; however they are different in the mean line or camber defined by the slope at the leading edge. It can be noted that the symmetric airfoils have good stall behavior and tend to exhibit high pitching moment coefficients. They also present a high drag and insensitive to roughness. Some of the common applications where they are used include helicopter blades, missiles and shrouded turbines in general aviation industry.

Table 1: NACA Airfoil series nomenclature

Sl. No.	NACA Airfoil	Thickness %	Maxim camber	Design lift coefficient
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1	0015	15	-	-
2	0024	24	-	-
3	4412	12	0.04, 0.4	-
4	66018	18	0.06, 0.6	-
5	16006	6	0.06, 0.6	-
6	64206	6	0.04, 0.4	0.2

Fig 2 & Fig 3 shows the NACA 00XX series profiles which produce different characteristics at range of angle of attack i.e. -1 deg to 3 deg respectively. The upper (suction) and lower (pressure) surfaces are both closer in case of NACA 0015 while NACA lower surface pressure coefficients are lower compared to upper surface in NACA 0024 as the angle of attack is changed. Further, with the change in onset flow velocity, i.e with increasing Reynolds number, the relative values for pressures on the surfaces show significant variances. Towards the trailing edge, the pressure gradient suddenly drops where the velocities at the upper & lower surfaces reach nearly the same limiting values per the *kutta condition*.

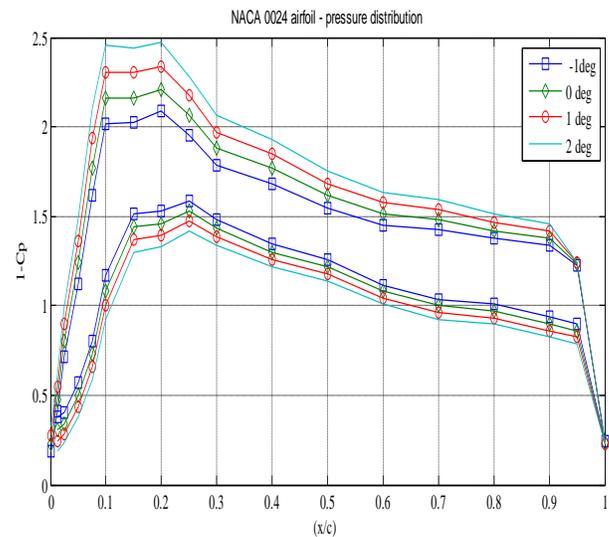


Figure 5: Pressure distribution of NACA 0024 airfoil

From fig 4 the NACA 4412 profile shows the significant high drag and low maximum lift coefficient and as the incidence angle is changed the upper and lower surface pressure values gap narrow. Further, at higher Reynolds number, or negative AoA [9] it shows the pressure reversal of upper (suction) and lower (pressure) surfaces can be observed. Fig 9 shows the comparison of pressure distribution indicate that the NACA 6 series exhibit high maximum lift coefficients compared with the 4 digit and modified 4 digit series.

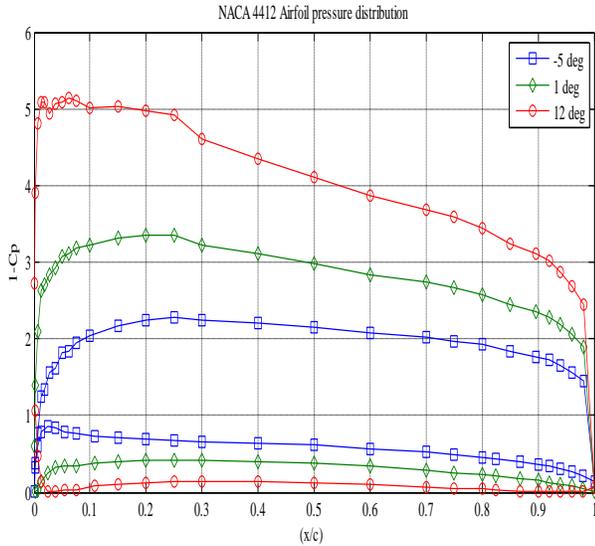


Figure 6: Pressure distribution of NACA 4412 airfoil

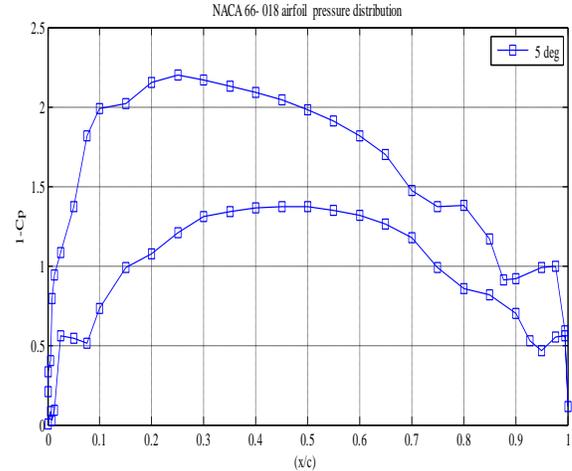


Figure 9: Pressure distribution of NACA 66-018 airfoil

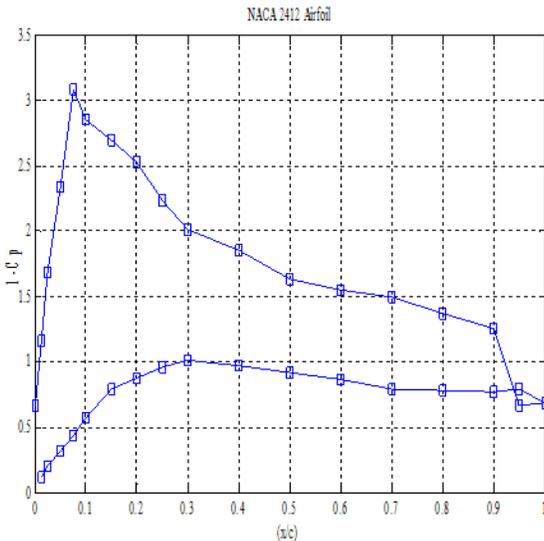


Figure7: Pressure distribution of NACA 2412 airfoil

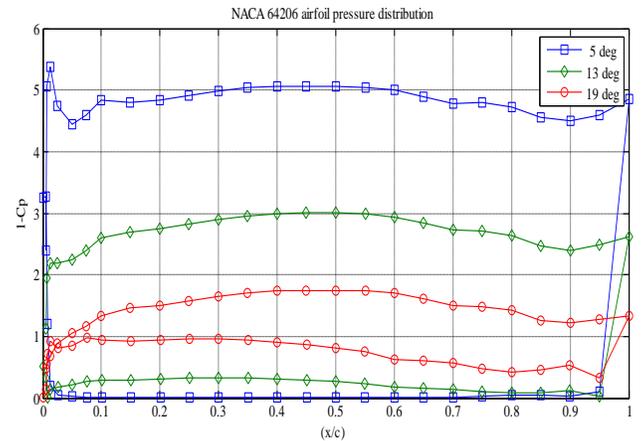


Figure 10: Pressure distribution of NACA 64-206 airfoil

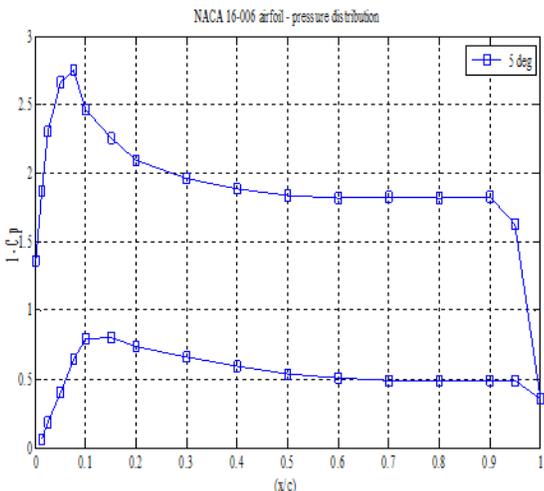


Figure 8: Pressure distribution of NACA 16-006 airfoil

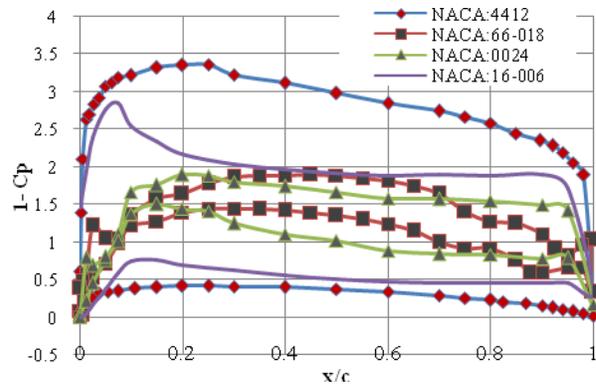


Figure 11 Comparison of pressure distribution for various NACA profiles using computational method.

The lift and drag characteristics were also computed using panel method by resolving the resulting force which is inclined at small angle with the AoA on the suction side of airfoil. The lift and drag can be expressed as

$$C_L = C_z \cdot \cos \alpha - C_x \cdot \sin \alpha \dots (10)$$

$$C_D = C_z \cdot \cos \alpha + C_x \cdot \sin \alpha \dots (11)$$

Where, C_z and C_x are the coefficients along the Y and X directions of the airfoil coordinate system. The values are obtained by pressure gradient coefficients of the upper and lower surfaces of airfoil and multiplying with the length of panel along the Y and X directions for each panel. The drag polar are plotted in fig 12 and 13 for the NACA 00XX airfoils. A *drag polar* is any mathematical function which relates the drag coefficient with some function of lift coefficient. Most often the drag coefficient is dependent upon the Reynolds number and Mach number and also expressed in terms of aspect ratio of wing.

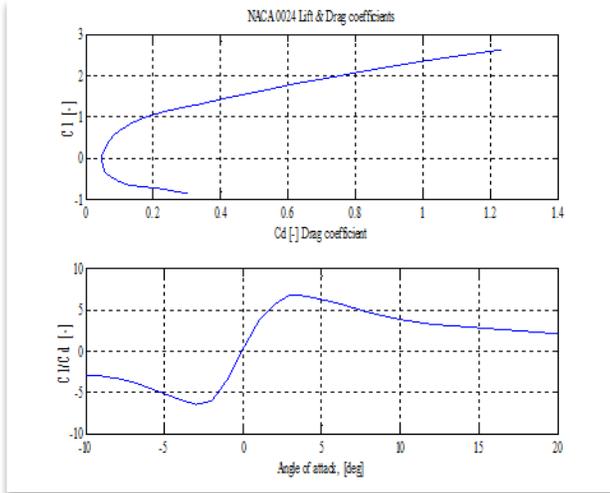


Figure 22: Lift and Drag Characteristics of NACA 0024 airfoil

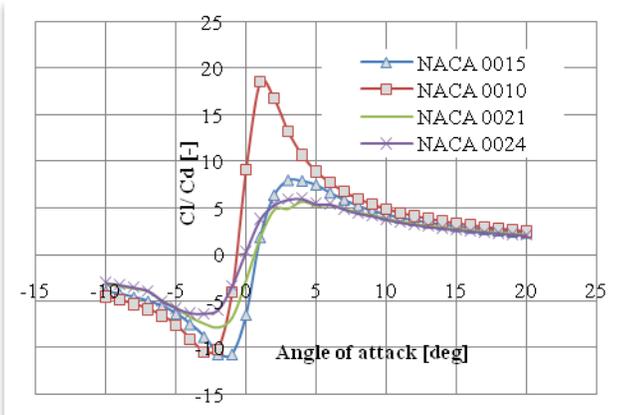


Figure 33: Comparison of CL/CD ratios of NACA 00XX series

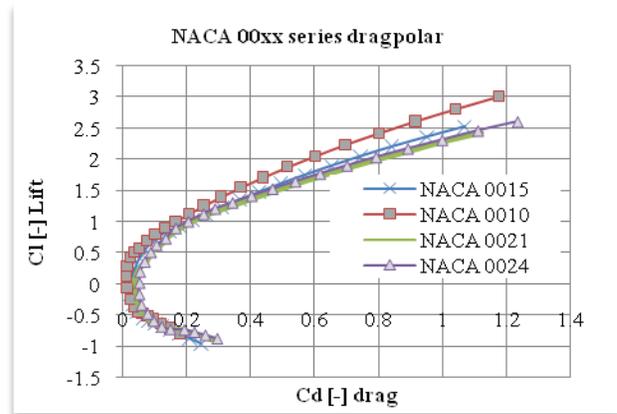


Figure 14: Comparison of NACA 00XX series - Drag polar

5. Conclusions

NACA airfoil study on pressure distribution, lift and drag attributes was performed using the computational panel method for 2D lifting flows. From the results this method is more useful when the computational requirements are low and with reduced time to validation of the results. It also predicts the aerodynamic characteristics with reasonable accuracy. The number of panels defined for the airfoil will affect the accuracy of the predicted values and its flow characteristics parameters. It overcomes the limitations present in the traditional methods developed for the flows over the non-lifting bodies.

6. References

- i. *Theory of Wing Sections, Including summary of airfoil data.* H. Abbot, A von Doenhoff. Dover publications Inc. New York.
- ii. *Aerodynamics for Engineering students,* Haughton, Carpenter, 6th Edition, Elsevier publications.
- iii. *Aerodynamics.* L J Clancy Sterling Book House.Indian edition 2006.
- iv. *Computer Program to obtain ordinates for NACA airfoils.* Charles L Ladson, Langley Research Center. Hampton, Virginia.
- v. *Theoretical and experimental data for NACA 6A series airfoil sections.* Lawrence K Loftin Jr. Report no 903.
- vi. *An overview of NACA 6 digit airfoil series characteristics with reference to airfoils for large wind turbine blades.* W.A Timmer. Delft university The Netherlands. AIAA. Jan 2009.
- vii. http://m-lig.ae.illinois.edu/ads/coord_database.html?
- viii. <http://www.mh-aerotoools.de/airfoils/velocitydistributions.htm>
- ix. *Ervin Bossanyi, Wind Energy Handbook,* Wiley Eastern Edition,