International Journal of Engineering Research
Volume No.8, Issue No.3, pp : 70-74
ISSN:2319-6890 (online),2347-5013(print)
1 July 2019

Effects of Chemical Reaction and Radiation on MHD Convective Casson Fluid Flow

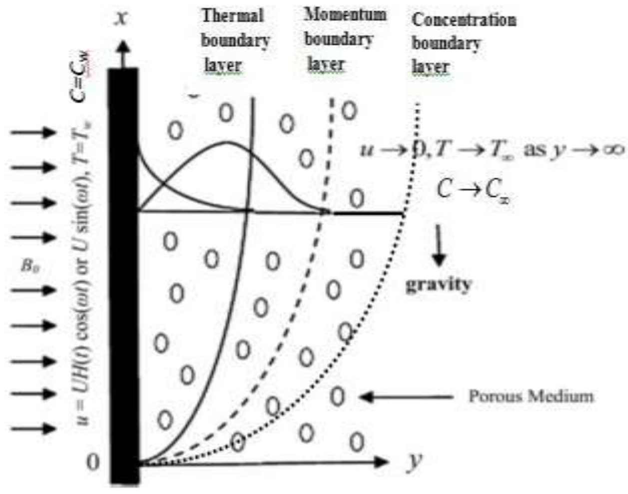
M. Umamaheswar[1], P. Chandra Reddy[1], M. Chengal Raju[2]
[1]Department of Mathematics, Annamacharya Institute of Technology and Sciences (Autonomous), Rajampet, India
Email: umasvu8@gmail.com; chandramsc01@gmail.com
[2]Department of Mathematics, JNTUA College of Engineering, Pulivendula, A.P., India
Email:mcrmaths@gmail.com

Abstract: ***This manuscript presents a detailed numerical study on the influence of radiation, radiation absorption and chemical reaction on unsteady magneto hydrodynamic free convective heat and mass transfer flow of a heat generating Casson fluid past an oscillating vertical plate embedded in a porous medium in the presence of constant wall temperature and concentration. The non dimensional governing equations along with the corresponding boundary conditions are solved using finite difference method numerically. Effects of various emerging flow parameters on velocity, temperature and concentration are presented graphically and analyzed. Expressions for skin-friction, Nusselt number and Sherwood number are also obtained. Concentration of Casson fluid increases for increasing values of Schmidt number and chemical reaction parameter.***

Keywords: Casson fluid, MHD, porous medium, heat and mass transfer, chemical reaction, radiation absorption and heat generation.

1. Introduction:

The studies on non-Newtonian fluids plays a vital role in many industries and so researchers are showing interest on these fluid flows in recent years. In general, non-Newtonian fluid is treated as elastic solid. Casson fluid is one of the non - Newtonian fluids. It can be defined as a shear thinning liquid which is supposed to have an infinite viscosity at zero rate of shear and a yield stress under which no flow occurs and zero viscosity at an infinite rate of shear. If yield stress greater than the shear stress is applied to the fluid, it behaves like solid. If yield stress less than the shear stress then the movement in the fluid starts. It is first invented by Casson in 1959. It is based on the structure of liquid phase and interactive behavior of solid of a two-phase suspension. Some examples of Casson fluid are Jelly, honey, tomato sauce and concentrated fruit juices. Human blood can also be treated as a Casson fluid in the presence of several substances such as fibrinogen, globulin in aqueous base plasma, protein, and human red blood cells. Many researchers considered the flow of Casson fluid and analyzed it under the influence of different physical parameters and also variety of boundary conditions. Animasaun (I) analyzed the effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction. Benazir et al. (II) studied magnetohydrodynamic Casson fluid flow over a vertical cone and flat plate with non-uniform heat source/sink. Chandra Reddy et al. (III) revealed on Casson fluid flow over a vertical porous plate under the existence of cross diffusion effects in conducting field. Cogley et al. (IV) considered a differential approximation for radiative transfer in a non-gray gas near equilibrium. Daba and Devaraj (V) considered unsteady hydromagnetic chemically reacting mixed convection flow over a permeable stretching surface with slip and thermal radiation. Dash et al. (VI) analyzed Casson fluid flow in a pipe filled with a homogeneous porous medium. Hayat et al. (VII) analyzed mixed convection flow of Casson nanofluid over a stretching sheet with convectively heated chemical reaction and heat source/sink. Hayat et al. (VIII) reported Soret and Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid. Hussanan (IX) studied unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Kandasamy and Pai (X) found and examined entrance region flow of Casson fluid in a circular tube. Kataria and Patel (XI) examined radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium. Khalid et al. (XII) considered and studied unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in porous medium.



Physical diagram of the problem

The tensor of the Casson fluid can be written as

$$\tau = \tau_0 + \mu\gamma^* \quad \text{or}$$

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases}$$

Where $\pi = e_{ij}\ e_{ij}$ and eij is the (i,j)th component of deformation rate, π is the product of the component of deformation rate with itself , π_c is the critical value of this product based on the non-Newtonian fluid, μ_B is the plastic dynamic viscosity of its fluid and is yield stress of the non-Newtonian fluid. Before forming the governing equations we have taken some assumptions that are unidirectional flow, one dimensional flow, free convection, rigid plate, incompressible flow, unsteady flow, non-Newtonian flow, oscillating vertical plate and viscous dissipation term in the energy equation is neglected. Considering the above assumptions, we have formed the following set of partial differential equations.

$$\rho\frac{\partial u'}{\partial t'} = \mu_\beta\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu\phi}{k_1}u' + \rho g\beta(T'-T_\infty) + \rho g\beta^*(C'-C_\infty) \qquad (1)$$

$$\rho C_p\frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + Q'(T'-T_\infty) + Q_1(C'-C_\infty) + \sigma B_0^2 u'^2 \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'\left(C'-C_\infty\right) \qquad (3)$$

Cogley et al. (1968) have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r}{\partial y'} = 4(T'-T_\infty)I \qquad \text{Where } I = \int K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$$

The initial and boundary conditions are

$$\left.\begin{array}{l} t'<0: u'=0, T'=T_\infty, C'=C_\infty \ \ \forall y'<0 \\ t'\ge 0: u'=u_0\sin(w't'), \\ \qquad T'=T_w, C'=C_w \ \ \text{at } y'=0 \\ u'\to 0, T'\to T_\infty, C'\to C_\infty \ \ \text{as } y'\to\infty \end{array}\right\} \qquad (4)$$

On introducing the following non-dimensional quantities

$$u=\frac{u'}{u_0},\ t=\frac{t'u_0^2}{\nu},\ y=\frac{y'u_0}{\nu},$$

$$\theta=\frac{T'-T_\infty}{T_w-T_\infty},\ C=\frac{C'-C_\infty}{C_w-C_\infty}$$

$$Gr=\frac{\nu g\beta(T_w-T_\infty)}{u_0^3}, Gc=\frac{\nu g\beta^*(C_w-C_\infty)}{u_0^3}$$

$$K=\frac{k_1u_0^2}{\phi\nu^2}, M=\frac{\sigma B_0^2\nu}{\rho u_0^2}, \Pr=\frac{\nu\rho C_p}{\kappa}$$

$$Ec=\frac{u_0^2}{C_p(T_w-T_\infty)}, Sc=\frac{\nu}{D}$$

$$Q=\frac{Q'\nu}{\rho C_p u_0^2}, R=\frac{4\nu I}{\rho C_p u_0^2}$$

$$\chi=\frac{Q_1\nu(C_w-C_\infty)}{\rho C_p u_0^2(T_w-T_\infty)}, Kr=\frac{Kr'\nu}{u_0^2}$$

In terms of the above non-dimension quantities, Equations (1)-(3) reduces to

$$\frac{\partial u}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - M u - \frac{1}{K}u + Gr\,\theta + Gm\,C \qquad (5)$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} - R\theta + Q\theta + \chi C + M\,Ec\,u^2 \qquad (6)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - Kr\,C \qquad (7)$$

The corresponding initial and boundary conditions are:

$$\left.\begin{array}{ll} t<0: u=0, T=0, C=0 & \text{for all } y<0 \\ t\ge 0: u=\sin(wt), \theta=1, C=1 & \text{at } y=0 \\ \quad u\to 0, T\to 0, C^*\to 0 & \text{as } y\to\infty \end{array}\right\} \qquad (8)$$

3. Method of solution:

Equations (5)-(7) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (8). However exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (5)-(7) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} = \left(1+\frac{1}{\gamma}\right)\left(\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right) - M\,u_{i,j} - \frac{1}{K}u_{i,j} + Gr\,\theta_{i,j} + Gc\,C_{i,j} \qquad (9)$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t} = \frac{1}{\Pr}\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right) - R\theta_{i,j} + Q\theta_{i,j} + \chi C_{i,j} + M\,Ec\left(u_{i,j}\right)^2 \qquad (10)$$

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t} = \frac{1}{Sc}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right) - Kr\,C_{i,j} \qquad (11)$$

Here, index i refer to y and j to time. The mesh system is divided by taking Δy = 0.04. From the initial condition in (8), we have the following equivalent:

$$u(i,0)=0,\ \theta(i,0)=0, C(i,0)=0\ \forall i \qquad (12)$$

The boundary conditions from (8) are expresse -difference form as follows

$$\begin{array}{l} u(0,j)=1, \theta(0,j)=1, C(0,j)=1\ \forall j \\ u(i_{\max},j)=\sin(w*(j-1)*\Delta t), \\ \theta(i_{\max},j)=1, C(i_{\max},j)=1\ \forall j \end{array} \qquad (13)$$

(Here i_{max} was taken as 201)

First the velocity at the end of time step viz, u (i, j+1), (i=1,201) is computed from (9) in terms of velocity, temperature and concentration at points on the earlier time-step. Then θ (i, j +1) is computed from (10) and C (i, j +1) is computed from (11). The procedure is repeated until
t = 0.05 (i.e. j = 500). During computation Δt was chosen as 0.0001.

Skin-friction:

The skin-friction in non-dimensional form is given by

$$\tau = -\left(1+\frac{1}{\gamma}\right)\left(\frac{du}{dy}\right)_{y=0}, where\ \tau^* = \frac{\tau}{\rho u_0^2}$$

Rate of heat transfer:

The dimensionless rate of heat transfer is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

Rate of mass transfer:

The dimensionless rate of mass transfer is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

4. Result and Discussion:

A numerical study has been carried out on the MHD flow of a Casson fluid. The effects of various physical parameters such as Prandtl number, heat source, radiation parameter and Schmidt number on temperature and concentration are discussed with help of graphs whereas Skin friction, Nusselt number and Sherwood are also discussed with the help of tables. Fig.1, depicts the effect of heat source on temperature. It is noticed that the temperature is increased by an increase in the heat source by the fluid. The central reason behind this effect is that the heat source causes an increase in the kinetic energy as well as thermal energy of the fluid. The momentum and thermal boundary layers get thinner in case of heat source fluids. Fig.2, demonstrates the effect of radiation parameter on temperature. It is observed that temperature decreases as radiation parameter increases. Fig.3 depicts the variations in temperature profile for different values of radiation absorption parameter. It is noticed that temperature increases as radiation absorption parameter increases. Influence of Schmidt number on concentration is shown in fig.4, from this figure it is noticed that concentration decreases with an increase in Schmidt number. Because, Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Therefore concentration boundary layer decreases with an increase in Schmidt number. Fig.5 indicates that, concentration profile decreases with an increase in Kr. Fig.6 displays the validation of our methodology. We compared the present results with the published results of Khalid et al. [1]. An excellent agreement is found in this comparison.

From table.1, we observed that the Nusselt number increase with increasing values of Prandtl number and radiation parameter while it decreases with increasing value of heat source and radiation absorption parameter. From table.2, we have seen that the Sherwood number increase with increasing values of Schmidt number and chemical reaction parameter.

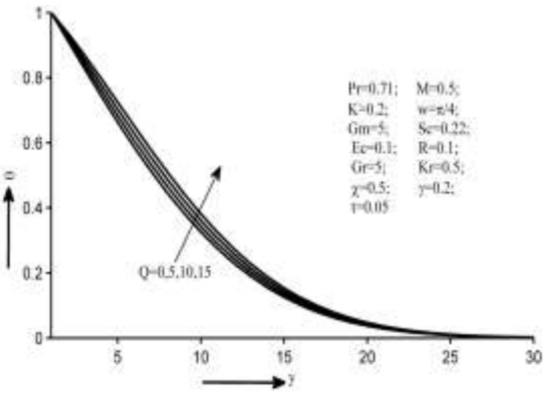


Fig. 1: Effect of heat source on temperature

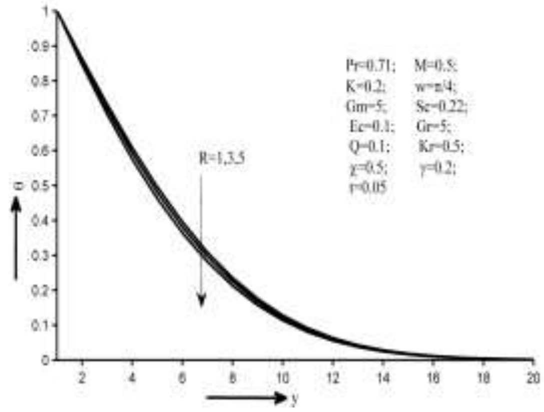


Fig. 2: Effect of radiation parameter on velocity

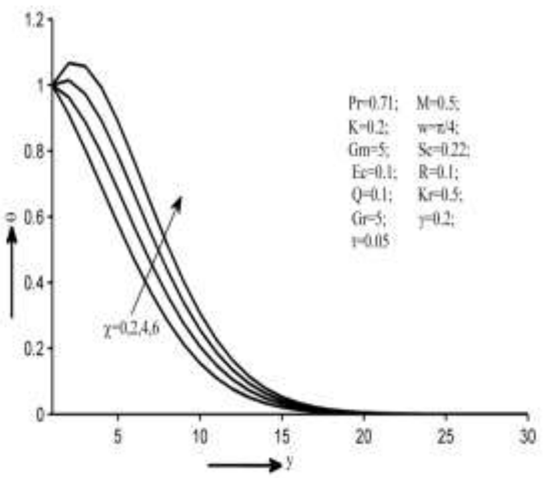


Fig. 3: Effect of radiation absorption parameter

ISSN:2319-6890 (online),2347-5013(print)
1 July 2019

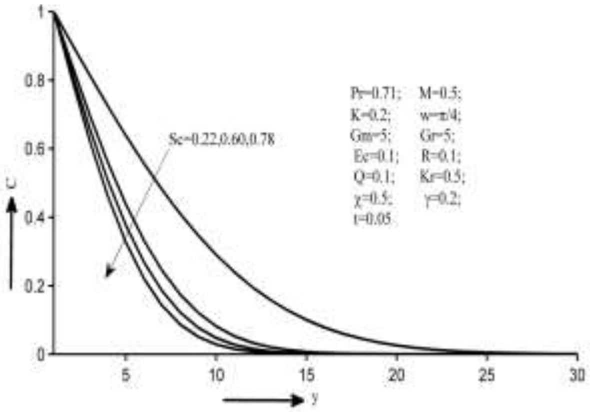


Fig. 4: Effect of Schmidt number on concentration

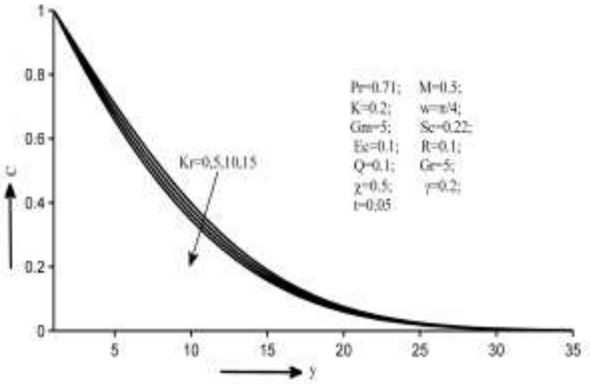


Fig. 5: Effect of Chemical reaction parameter on concentration

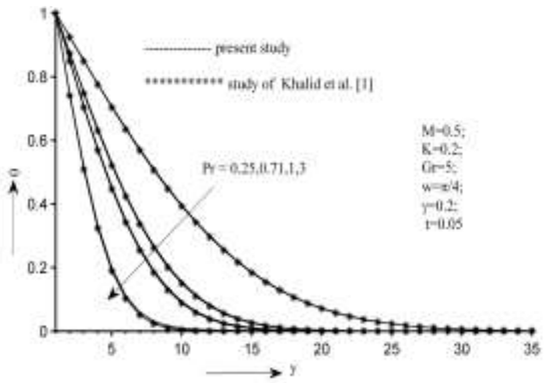


Fig 6: Comparison of present study with the study of Khalid et al. (2015) in the absence of Gm, Q, R, Ec, χ, Sc, Kr.

Table.1: Variations in Nusselt number

Pr	Q	R	χ	Nu
0.71	0.8	0.8	0.1	4.4626
1	0.8	0.8	0.1	5.3128
3	0.8	0.8	0.1	8.6469
7.1	0.8	0.8	0.1	11.1283
0.71	0.1	0.8	0.1	5.3356
0.71	0.3	0.8	0.1	5.3291
0.71	0.5	0.8	0.1	5.3256
0.71	1	0.8	0.1	5.3024
0.71	0.8	0.5	0.1	5.4492
0.71	0.8	1	0.1	5.6524
0.71	0.8	2	0.1	5.8672
0.71	0.8	0.8	1	4.4124
0.71	0.8	0.8	2	3.5264
0.71	0.8	0.8	3	3.0564
0.71	0.8	0.8	4	2.5648

Table.2: Variations in Sherwood number

Sc	Kr	Sh
0.22	0.8	3.5484
0.60	0.8	4.6542
0.78	0.8	4.8492
0.96	0.8	5.5321
0.22	0.1	3.5234
0.22	0.3	3.6134
0.22	0.5	3.7568
0.22	0.9	3.8829

5. Conclusion: The following are the conclusions of this manuscript.

1. Temperature of the Casson fluid increases with increasing values of R, Q and χ whereas reverse trend is seen in the case of R and Pr.
2. Concentration of Casson fluid increases for increasing values of Sc and Kr.
3. Nusselt number increase with increasing values of Prandtl number and radiation parameter while it decreases with increasing value of heat source and radiation absorption parameter.
4. Sherwood number increase with increasing values of Schmidt number and chemical reaction parameter.

References:

i. *Animasaun I (2015). Effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction, J. Nigerian Math. Soc., 34, 11-31.*

ii. *Benazir A. J., Sivaraj R, Makinde O.D (2016). Unsteady Magnetohydrodynamic Casson fluid flow over a vertical cone and flat plate with non-uniform heat source/sink, International Journal of Engineering Research in Africa, 2, 69-83.*

iii. *Chandra Reddy P, Raju M.C., Harinath Reddy S, Raju G.S.S (2018). Casson fluid flow over a vertical porous plate under the existence of cross diffusion effects in conducting field, International Journal of Advanced Scientific Research and Management, 3 (11) 380-387.*

iv. *Cogley A.C, Vincenti W.G., Gill S.E (1968). Differential approximation for radiative transfer in a non-gray gas near equilibrium, AIAA J., 6, 551–553.*

v. *Daba M, Devaraj P (2016). An unsteady hydromagnetic chemically reacting mixed convection flow over a permeable stretching surface with slip and thermal radiation, Journal of the Nigerian Mathematical Society, 35, 245-256.*

vi. *Dash R, Mehta K, Jayaraman G (1996). Casson fluid flow in a pipe filled with a homogeneous porous medium, Int. J. Eng. Sci., 34, 1145-1156.*

vii. *Hayat T, Bilal Ashraf M, Shehzad S.A., Alsaedi A (2015). Mixed convection flow of Casson nanofluid over a stretching sheet with convectively heated chemical reaction and heat source/sink, Journal of Applied Fluid Mechanics, 8(4), 803-813.*

viii. *Hayat T, Shehzad S, Alsaedi A (2012). Soret and Dufour effects on MHD flow of Casson fluid, Appl. Math. Mech., 33, 1301-1312.*

ix. *Hussanan A, Salleh M.Z., Tahar R. M., Khan I (2014). Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating, PLoS One, 9, 1-9.*

x. *Kandasamy A, Pai R.G (2012). Entrance region flow of Casson fluid in a circular tube, Appl. Mech Mater, 698–706.*

xi. *Kataria H.R., Patel H.R (2016). Radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium, Alexandria Engineering Journal, 55(1), 583-595.*

xii. *Khalid A, Khan I, Khan A (2015). An unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in porous medium, Engineering Science and Technology, an International Journal, 18(3), 309-317.*