

MHD Flow of Jeffrey Fluid past an Inclined Deformable Porous Channel in the Presence of Slip Conditions

Ananda Reddy Narravula

Department of Mathematics, A.I.T.S., Rajampet, YSR dist., A.P., INDIA

Email : anandareddy@gmail.com

Abstract : *In the present study, numerical solution for MHD flow of Jeffrey fluid through an inclined deformable porous channel with slip conditions is obtained. The governing partial differential equations are converted to ordinary non-linear differential equations with the help of dimensional-less quantities. These equations are solved by using shooting technique. The influence of governing parameters on the fluid velocity, the solid displacement and the concentration are represented in the form of graphs. The present results have been in good agreement with prevailing studies under some distinct cases.*

Key words: Jeffrey parameter; MHD flow; slip conditions.

Introduction

The study of heat and mass transfer in fluid flows through porous media is the current area of research interest due to its widespread applications in engineering, biology and medicine. The non-Newtonian fluid flow past porous media occurs in many industrial situations and has got several important scientific and engineering applications such as flow through packed beds and ion-exchange beds, energy extraction from the geothermal regions, solid filtration from liquids. The Mathematical modeling of deformable porous media have interesting applications in understanding the blood flow in the tissue region of the blood vessel, synovial fluid flow in articular cartilage and erythrocyte deformability etc. In view of these applications Asghar *et al.* [1] studied flow and heat transfer analysis in a deformable channel. Syed Tauseef Mohyud-Din *et al.* [2] developed flow of a radioactive Casson fluid through a deformable asymmetric porous channel. Madhusudhana Rao *et al.* [3] reported slip effects on MHD three dimensional flow of Casson fluid over an exponentially stretching surface. Sreenadh *et al.* [4] analyzed MHD Couette flow of a Jeffrey fluid over a deformable porous layer. Sudhakara *et al.* [5] discussed effect of heat transfer on free surface flow of a Jeffrey fluid over a deformable permeable bed. Barry *et al.* [6] investigated on Fluid flow over a thin deformable porous layer. Dariusz Gawin *et al.* [7] discussed coupled heat, water and gas flow in deformable porous media. Entropy generation analysis for MHD flow through a vertical deformable porous layer was developed by Sreenadh *et al.* [8]. Narla *et al.* [9] reported on modeling transient MHD peristaltic pumping of electroconductive viscoelastic fluids through a deformable curved channel. Numerical solutions for unsteady flows of a MHD Jeffrey fluid between parallel plates

through a porous medium was discovered by Ramesh *et al.* [10]. Effects of thermal radiation and magnetohydrodynamics on Ree-Eyring fluids flows through porous medium with slip boundary conditions was developed by Ramesh *et al.* [11]. Venkata Satya Narayana *et al.* [12] studied numerical study of a Jeffrey fluid over a porous stretching sheet with heat source/sink. Gopi Krishna *et al.* [13] investigated an entropy generation on viscous fluid in the inclined deformable porous medium. Effects of viscous dissipation and flow work on forced convection in a channel filled by a saturated porous medium were analysed by Nield *et al.* [14].

Numerical solution for hydromagnetic flow of Jeffrey fluid over an inclined deformable porous channel with slip conditions is obtained in the present paper. The governing equations of the fluid velocity, the solid displacement and the concentrations are solved numerically with shooting technique. The effects of governing parameters on the fluid velocity, the solid displacement and the concentrations are shown graphically.

Mathematical Formulation of the Problem

Consider, the steady hydromagnetic flow of Jeffrey fluid over an inclined deformable porous channel with slip conditions. The barring plates are moving with distinct velocities U_1 and U_2 respectively. The x – axis be taken along the flow direction and y – is perpendicular to it. The plates are maintained at different temperatures T_w , T_0 and the distinct concentrations C_w , C_0 respectively. A pressure gradient $\frac{\partial p}{\partial x}$ is applied producing an axially directed flow.

The governing equations of the momentum, the solid displacement, the temperature and the concentration equations are:

$$\frac{2\mu_a}{1+\lambda_1} \frac{\partial^2 v}{\partial y^2} - \phi \frac{\partial p}{\partial x} - \sigma B_0^2 v + \rho g \sin \theta - K v = 0 \quad (1)$$

$$\mu \frac{\partial^2 u}{\partial y^2} - (1-\phi) \frac{\partial p}{\partial x} + \rho g \sin \theta + K v = 0 \quad (2)$$

$$K_0 \frac{\partial^2 T}{\partial y^2} + \frac{2\mu_a}{1+\lambda_1} \left(\frac{\partial v}{\partial y} \right)^2 + Q_0 = 0 \quad (3)$$

$$D_B \frac{\partial^2 C}{\partial y^2} - R(C - C_0) = 0 \quad (4)$$

The governing boundary conditions are:

$$\left. \begin{aligned} v = U_1 + L_1 \frac{\partial v}{\partial y}, u = 0, T = T_0 + L_2 \frac{\partial T}{\partial y}, C = C_0 + L_3 \frac{\partial C}{\partial y} \text{ at } y = -h \\ v = U_2, u = 0, T = T_1, C = C_1 \text{ at } y = h \end{aligned} \right\} \quad (5)$$

Where μ_a is the apparent viscosity of the fluid in the porous material, K is the drag coefficient, μ is the Lamé constant, θ is the angle of inclination, K_0 is the thermal conductivity, v is the fluid velocity, Q_0 is the constant heat source/sink, ϕ is the volume fraction of the fluid, u is the solid displacement, ρ is the fluid density, g is the gravity, $\frac{\partial p}{\partial x}$ is the pressure gradient, σ is the electrical conductivity, B_0 is the strength of the magnetic field, U is the average velocity and λ_1 is the Jeffrey parameter, L_1 is the velocity slip parameter, L_2 and L_3 are the temperature and concentration slip parameters, D_B is the thermo diffusion coefficient, T and C are the temperature and the concentration, C_1 is the constant concentration at the walls, C_0 ambient concentration, R is the chemical reaction.

Now introducing the non-dimensional quantities are as follows:

$$\left. \begin{aligned} Fr = \frac{U}{gh}, U_1^* = \frac{U_1}{U}, U_2^* = \frac{U_2}{U}, y^* = \frac{y}{h}, v^* = \frac{v}{U}, x^* = \frac{x}{h}, \gamma = \frac{Rh^2}{D_B} \\ T^* = \frac{T - T_0}{T_w - T_0}, C^* = \frac{C - C_0}{C_w - C_0}, p^* = \frac{hp}{2\mu_a U}, u^* = \frac{u\mu}{2\mu_a U}, \frac{dp}{dx} = P, \delta = \frac{Kh^2}{2\mu_a} \\ Re = \frac{\rho U h}{2\mu_a}, \beta = Q_0 \left(\frac{h^2}{K_0(T_w - T_0)} \right), Br = \frac{2\mu_a U^2}{K_0(T_w - T_0)}, M^2 = \frac{\sigma B_0^2 h^2}{\mu} \end{aligned} \right\} \quad (6)$$

Using equation (6) in equations (1)-(5) and after neglecting the asterisks (*) takes the following equations are:

$$\frac{1}{1 + \lambda_1} \frac{d^2 v}{dy^2} - \phi P - \frac{Re}{Fr} \sin \theta - (\delta + M^2)v = 0 \quad (7)$$

$$\frac{d^2 u}{dy^2} - (1 - \phi)P - \frac{Re}{Fr} \sin \theta - \delta v = 0 \quad (8)$$

$$\frac{d^2 \theta}{dy^2} + \frac{Br}{1 + \lambda_1} \left(\frac{dv}{dy} \right)^2 + \beta = 0 \quad (9)$$

$$\frac{d^2 \phi}{dy^2} - \gamma \phi = 0 \quad (10)$$

The boundary conditions are

$$\left. \begin{aligned} v = U_1 + \alpha_1 \frac{dv}{dy}, u = 0, \theta = \alpha_2 \frac{d\theta}{dy}, \phi = \alpha_3 \frac{d\phi}{dy} \text{ at } y = -1 \\ v = U_2, u = 0, \theta = 1, \phi = 1 \text{ at } y = 1 \end{aligned} \right\} \quad (11)$$

Where λ_1 is the Jeffrey parameter, ϕ is the volume fraction of the fluid, Re is the Reynolds number, Fr is the Froude number, θ is the angle of inclination, δ is the viscous drag coefficient, M is the magnetic parameter, P is the pressure gradient, Br is the Brinkman number, β is the constant heat source parameter, U_1 and U_2 are the lower and upper plate velocities γ is the chemical reaction parameter, $\alpha_1 = \frac{L_1}{h}$ is the velocity slip parameter, $\alpha_2 = \frac{L_2}{h}$ and $\alpha_3 = \frac{L_3}{h}$ are the temperature and the concentration slip parameters.

Results and Discussion

In this paper, MHD flow of Jeffrey fluid over a deformable porous channel with slip conditions is studied. The solid displacement, the fluid velocity, the temperature and the concentration are solved by using Runge-Kutta fourth order along with shooting technique. The effects of governing parameters on the solid displacement, the fluid velocity, the temperature and the concentration are shown with the help of the graphs.

From Figures 1- 4, it has been noticed that the fluid velocity $v(y)$ and the solid displacement $u(y)$ are enhance for higher values of moving velocities U_1 and U_2 . The variation of magnetic parameter M on the fluid velocity $v(y)$ and the solid displacement $u(y)$ are explained in Figures 5 and 6. They reveal that both the fluid velocity and the solid displacement are decrease for enhance magnetic parameter. This decreasing effect in the fluid velocity and the solid displacement are an established result because the magnetic field acts in the transverse direction to the flow and magnetic force resists the flow. Figures 7 and 8 are represent the influence of Jeffrey parameter λ_1 on the fluid velocity $v(y)$ and the solid displacement $u(y)$. We found that both the fluid velocity and the solid displacement are enhancing for higher values of Jeffrey parameter. The impacts of the fluid velocity slip parameter α_1 , the temperature slip parameter α_2 and the concentration slip parameter α_3 on the fluid velocity $v(y)$, the temperature $\theta(y)$ and the concentration $\phi(y)$ are depicted in Figures 9-11. We reveal that the fluid velocity reduces with increasing the fluid

velocity slip parameter and the opposite behavior observed in the temperature and the concentration for higher values of the temperature slip parameter and the concentration slip parameter.

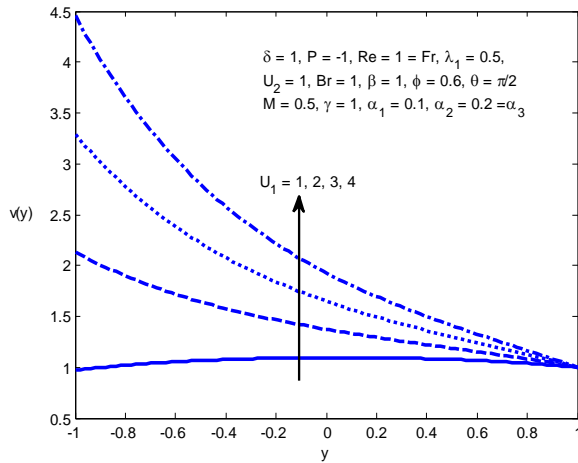


Figure 1. The impact of U_1 on the fluid velocity $v(y)$

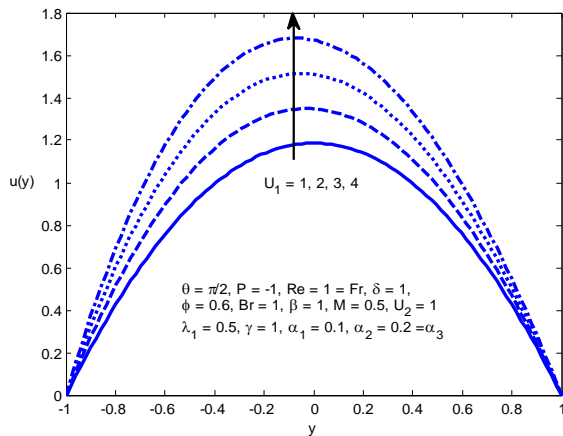


Figure 2. The impact of U_1 on the solid displacement $u(y)$

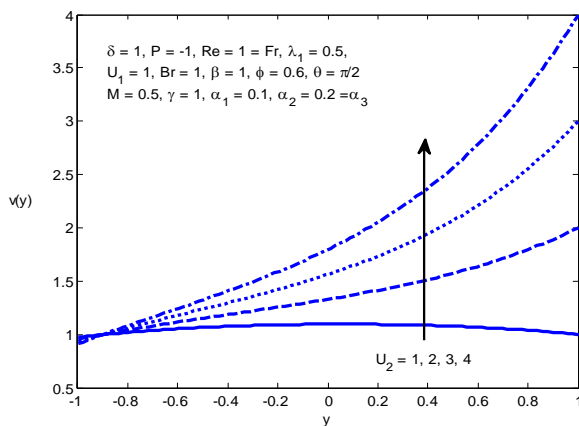


Figure 3. The impact of U_2 on the fluid velocity $v(y)$

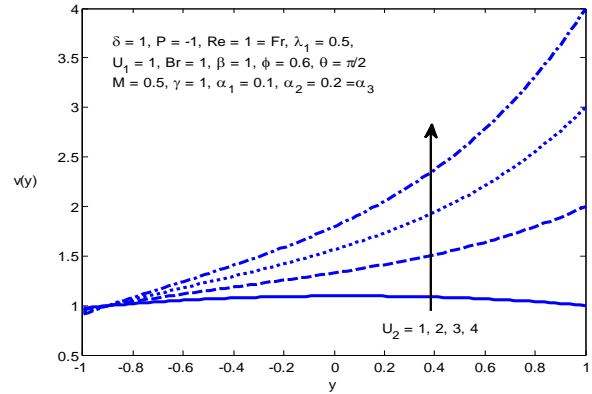


Figure 4. The impact of U_2 on the solid displacement $u(y)$

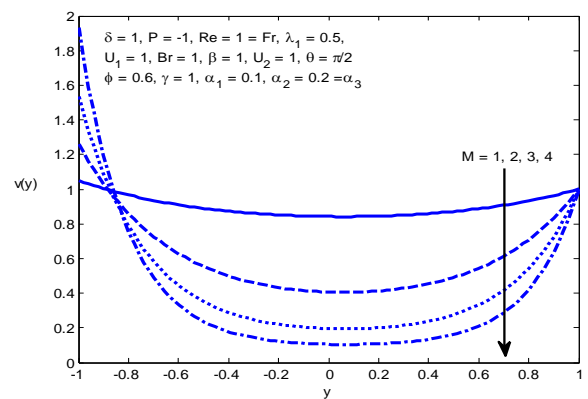


Figure 5. The impact of M on the fluid velocity $v(y)$

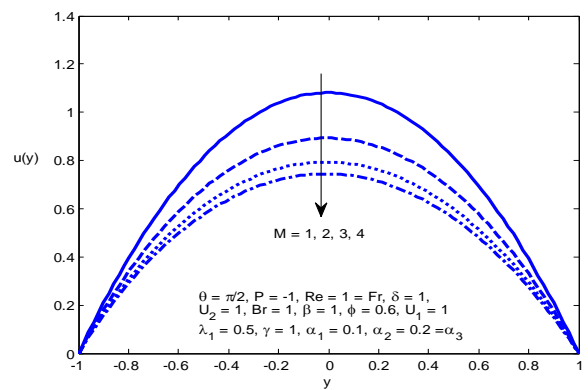


Figure 6. The impact of M on the solid displacement $u(y)$

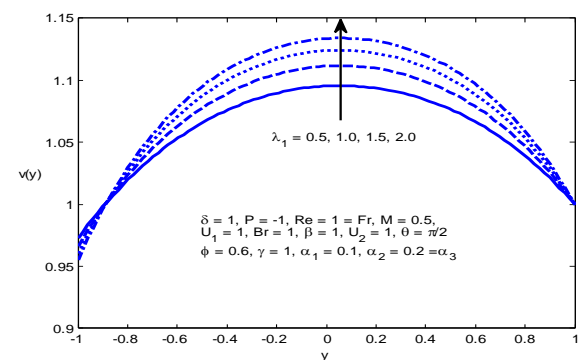


Figure 7. The impact of λ_1 on the fluid velocity $v(y)$

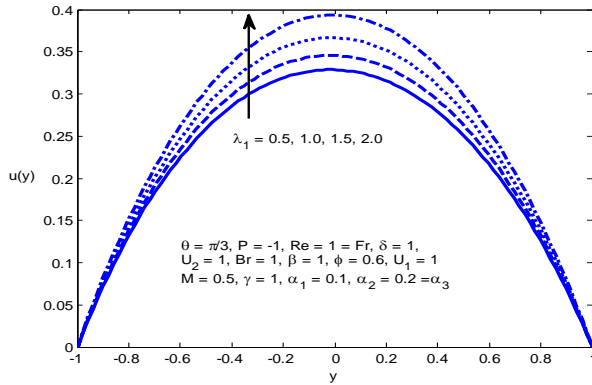


Figure 8. The impact of λ_1 on the solid displacement $u(y)$

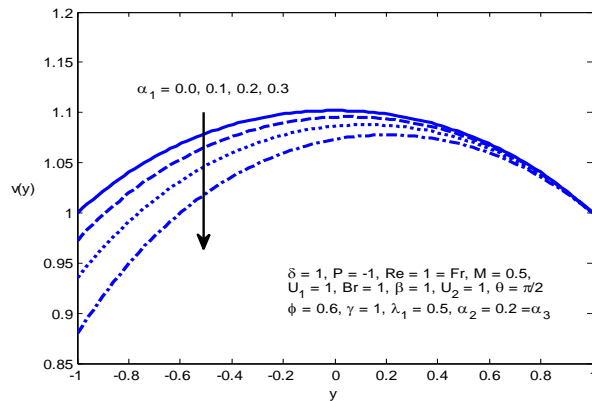


Figure 9. The impact of α_1 on the fluid velocity $v(y)$

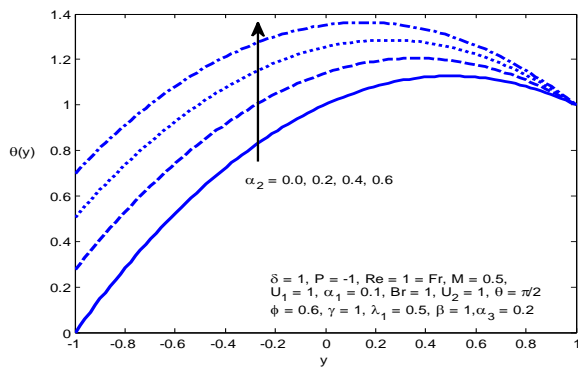


Figure 10. The impact of α_2 on the temperature $\theta(y)$

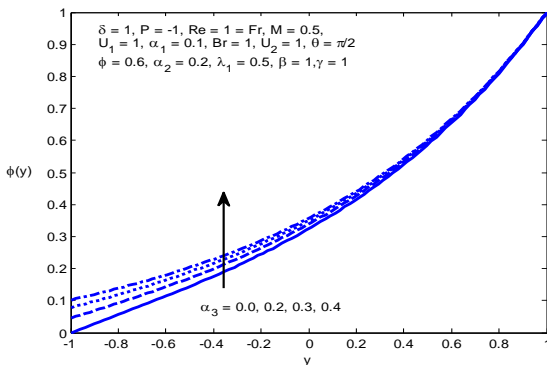


Figure 11. The impact of α_3 on the concentration $\phi(y)$

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