ISSN: 2319-6890 (online),2347-5013(print)
1 Jan. 2020

Thermal Radiation and Thermophoresis Effects on Steady MHD Free Convection Flow of a Micropolar Fluid through a Porous Medium with Variable Heat and Mass Flux Boundary Conditions

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Abstract: ***In this paper, the combined effects of thermal radiation and thermophoresis on steady magneto hydrodynamic free convection flow of a micro polar fluid in the presence variable heat and mass fluxes are taken into account is considered. The governing non linear partial differential equations of the problem are transformed into a system of nonlinear ordinary differential equations through appropriate similarity transformation and Runge–Kutta Fourth order with shooting technique method. The effects of various physical parameters on the dimensionless velocity, microrotation, temperature, and concentration, local skin friction coefficient, local Nusselt number and local Sherwood number are tabulated and discussed.***

Keywords: Thermal Radiation, Thermophoresis, MHD, Micropolar fluid, Porous Medium, Heat and Mass flux.

I. Introduction

The theory of micro polar has received enormous attentions during the recent years since the traditional Newtonian fluids cannot specifically depict the feature of fluid with suspended particles, polar fluids, suspension solutions, liquid crystals, colloidal solutions and fluid containing small additives. Physically, micropolar fluids may present the non-Newtonian fluids consisting of short rigid cylindrical elements or dumb-bell molecules, polymer fluids, fluids suspensions and animal blood. The existence of dust or smoke particular in a gas may also be modeled using micro polar fluid dynamics. Cogley et al. [i] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Kim and Fodorov [ii] considered the case of mixed convection flow of a micropolar fluid past a semi-infinite, steadily moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation. The transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate was studied by Makinde [iii]. Ibrahim et al. [iv] discussed the case of mixed convection flow of a micropolar fluid past a semi infinite, steady moving porous plate with varying suction velocity normal to the plate in presence of thermal radiation and viscous dissipation. Rahman and Sattar [v] studied transient convective heat transfer flow of a micropolar fluid past a continuously moving vertical porous plate with time dependent suction in the presence of radiation.

Most of the real time industrial processes involve heat and mass transfer. Heat or mass flux must be removed, added or moved from one stream process to another. In many practical situations occur in which the hot surface is subject to a constant heat flux instead of being at a prescribed temperature. Dutta et al. [vi] first investigated the effect of uniform heat flux on the temperature field in case of flow due to a stretching sheet. Simillar studies are found in [vii-x]. Most of previous works are not studied heat and mass transfer MHD free convective flow of micropolar fluid through a porous medium with heat and mass fluxes in the presence of the thermophoresis. Hence, in the present work, we have performed a numerical investigation on the combined effects of thermal radiation and thermophoresis on steady magnetohydrodynamic free convective heat and mass transfer flow of a micropolar fluid past a past a vertical porous plate with heat and mass flux boundary conditions.

II. Mathematical analysis

Let us, consider a steady two-dimensional MHD free convective flow of viscous incompressible electrically conducting fluid past a semi-infinite permeable inclined flat plate, while a magnetic field of uniform strength B_0 is applied in the *y*-direction which is normal to the flow direction. Fluid suction is imposed at the plate surface and the suction hole size is taken to be constant. The temperature of the surface is held uniform at T_w which is higher than the ambient temperature T_∞ .The Roseland approximation is used to describe the radioactive heat flux in the *x*-direction which is considered negligible in comparison to the *y*-direction. The effects of thermophoresis are being taken into account to help in the understanding of the mass deposition variation on the surface. Under the above assumptions, the governing equations for this problem can be written as:

(i) Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

(ii) Momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_a\frac{\partial^2 u}{\partial y^2}+\frac{S}{\rho}\frac{\partial N}{\partial y}+g\beta_T 2\left(T-T_\infty\right)$$
$$+g\beta_C\left(C-C_\infty\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\nu_a}{K'}(u-U_\infty)-\frac{b}{K'}(u-U_\infty)^2 \tag{2}$$

(iii) Angular momentum:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\nu_s}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{S}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right)=0 \tag{3}$$

(vi) Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{4}$$

(v) Concentration:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}(V_T C) \tag{5}$$

where u and v are the velocity components in the *x*, y directions, $\nu_a=(\mu+s)/\rho$ is the apparent kinematic viscosity, g is the acceleration due to gravity, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β_T and β_C are the coefficient of thermal and volumetric expansions , respectively, $v_s=(\mu+s/2)\,j$ is the microrotation viscosity or spin-gradient viscosity, j is the micro-inertia density, T is the temperature of thermal boundary layer fluid , C is the concentration of the fluid in the boundary layer, T_∞ is the temperature far away from the plate, C_∞ is the concentration of the solute far away from the plate, U_∞ is the velocity of the fluid far away from the plate, σ is the electrical conductivity, B_0 is the magnetic induction, k is the fluid thermal conductivity, ρ is the fluid density, C_p is the specific heat at constant pressure, q_r is the radiative heat flux in the *y*-direction, μ is the dynamic viscosity, D is the molecular diffusivity of the species concentration and V_T is the thermophoretic velocity.

The boundary conditions for the model are as follows

$$u=0,\ v=\pm v_w(x),\ N=-n\frac{\partial u}{\partial y},$$
$$\frac{\partial T}{\partial y}=-\frac{q_m}{k},\ \frac{\partial C}{\partial y}=-\frac{m}{D_m}\quad at\ y=0 \tag{6}$$
$$u\to U_\infty,\ N\to 0,\ T\to T_\infty,\ C\to C_\infty\ \text{as}\ y\to\infty$$

By using the Rosseland approximation, the radiative heat flux in the y' direction is given by

$$q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial Y} \tag{7}$$

where σ_1 is the Stefan-Boltzmann constant and k_1 is the mean absorption coefficient.

Assuming that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in Taylor series about the free stream temperature T_∞ to yield

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{8}$$

where the higher-order terms of the expansion are neglected.
By using (6) and (7), Eq. (4) gives

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_e}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

Now the thermophoretic velocity V_T , which appears in the Eq. (4) can be written as

$$V_T=-kv\frac{\nabla T}{T_{ref}}=\frac{-kv}{T_{ref}}\frac{\partial T}{\partial y} \tag{10}$$

A thermophoretic parameter τ can be defined as follows

$$\tau=\frac{-k(T_w-T_\infty)}{T_r} \tag{11}$$

Typical values of τ are 0.01, 0.05 and 0.1 corresponding to approximate values of $-k(T_w-T_\infty)$ equal to 3K, 15K and 30 K for a reference temperature of $T_r=300K$.

III. Similarity transformation

The solutions of the governing equations are obtained by introducing the following non-dimensional variables:

$$\eta=\left(\frac{U_\infty}{2\nu_a x}\right)^{1/2}y,\ u=U_0\,f'(\eta),\ \theta(\eta)=\frac{T-T_\infty}{Q/k}\sqrt{\frac{U_\infty}{2\nu_a x}},$$
$$\phi(\eta)=\frac{C-C_\infty}{m/D_m}\sqrt{\frac{U_\infty}{2\nu_a x}},\ \psi=(2\nu_a U_\infty x)^{1/2}f(\eta), \tag{12}$$
$$v=-\sqrt{\frac{2\nu_a U_\infty}{x}}\left[f(\eta)-\eta f'(\eta)\right],\ N=\left(\frac{U_\infty^3}{2\nu_a x}\right)^{1/2}g(\eta)$$

If the dimensional stream function $\psi(x,y)$ then $u=\frac{\partial\psi}{\partial y}$ and

$$u=-\frac{\partial\psi}{\partial x}. \tag{13}$$

The continuity equation is automatically satisfied and the system of Eqs. (2), (3), (5) and (9) becomes:

$$f'''+ff''+Kg'+Gr\theta+Gc\phi-Mf'-2\lambda(f'-1)$$
$$-Fs(f'-1)^2=0 \tag{14}$$

$$G_2 g''-2G_1(2g+f'')+f'g+fg'=0 \tag{15}$$

$$(3R+4)\theta''+3R\Pr f\theta'+3R\Pr f'\theta=0 \tag{16}$$

$$\phi''+Sc(f-\tau\theta')\phi'-Sc\tau\theta''\phi=0 \tag{17}$$

The primes mean differentiation with respect to η , $M=\frac{\sigma B_0^2 2x}{\rho U_\infty}$ is the is the magnetic field parameter,

$Gr=g\beta_T\sqrt{\frac{\nu_a(2x)^3}{U_\infty^5}}\frac{Q}{k}$ is the local thermal Grashof number,

$Gc=g\beta_c\sqrt{\frac{\nu_a(2x)^3}{U_\infty^5}}\frac{m}{D_m}$ is the local solute Grashof number, $K=\frac{s}{\rho\nu_a}$ is the coupling parameter, $\lambda=\frac{1}{Da}$ is the Darcy parameter, Da = is the modified Darcy number, $Fs=\frac{bx}{K'}$ is the modified Forchheimer number, $G_1=\frac{sx}{\rho jU_\infty}$ is the vortex viscosity parameter, $G_2=\frac{\nu_s}{\rho j\nu_a}$ is the spin gradient viscosity parameter, $G=\frac{G_1 U_0}{2\nu x}$ is the micro-rotation parameter, $\Pr=\frac{\rho\nu c_p}{k}$ is the Prandtl number, $R=\frac{kk_e}{4\sigma_s T_\infty^3}$ is the thermal radiation parameter, $Sc = \frac{\nu}{D_m}$ is the Schmidt number, parameter.

The transformed boundary conditions (6) are given by

$$f(0) = \pm f_w,\ f'(0) = 0,\ g(0) = 0,\ \theta(0) = -1,\ \phi(0) = -1$$
$$f(\infty) = 0,\ g(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0$$
(18)

Where $f_w = -v_w(x)\sqrt{\frac{2x}{\vartheta U_0}}$ is the permeability of the porous surface (positive for suction and negative for injection).

The physical quantities of interest are the local skin friction coefficient, the wall heat transfer coefficient (or the local Nusselt number) and the wall deposition flux (or the local Stanton number) which are defined as respectively where the skin friction C_f, the heat transfer $q_w(x)$ and the mass transfer Sh_x from the wall are given by

$$C_f \, \mathrm{Re}_x^{-1/2} = \frac{\tau_w}{(1/2)\rho U_o^2} = 2f''(0),\ \tau_w = \mu\left(\frac{du}{dy}\right)_{y=0} \quad (19)$$

The equation defining the plate couple stress is

$$M_w = \nu_s\left(\frac{\partial N}{\partial y}\right)_{y=0}$$
(20)

The dimensionless couple stress is defined by

$$M_s = \frac{M_w}{1/2\,\rho \nu_a U_\infty} = \frac{G_2 K}{G_1} g'(0)$$
(21)

Thus the local couple stress in the boundary layer is proportional to $g'(0)$.

From the temperature field, we can study the rate of heat transfer which is given by

$$Nu_x \, \mathrm{Re}_x^{-1/2}\left(\frac{3R}{3R+4}\right) = \frac{q_w(x)}{\lambda(T_w - T_\infty)} = -\frac{1}{2}\theta'(0);$$
$$q_w(x) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma_1}{3k_1}\left(\frac{\partial T^4}{\partial y}\right)_{y=0} \quad (22)$$

From the concentration field, we can study the rate of mass transfer which is given by

$$Sh_x St \, \mathrm{Re}_x^{-1/2} = -\frac{J_s}{U_0 C_\infty};\ J_s = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\phi'(0) \quad (23)$$

where $\mathrm{Re}_x = U_0 x/\nu$ the local Reynolds number.

IV. Method of solution:

The system of ordinary differential Eqs. (14) – (17) subject to the boundary conditions (18) are solved numerically using Runge – Kutta fourth-order with shooting iteration technique. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The results are presented graphically in Figs. 1–5 and conclusions are drawn for flow field and other physical quantities of interest that have significant effects.

V. Results and discussion

The Eqs. (11)–(13) constitute highly non-linear coupled boundary value problem of third and second order. Thus we have used the shooting iteration technique with Runge–Kutta fourth-order integration algorithm. For numerical results we considered the non dimensional parameter values as M =1.0, K=0.01, *Gr=2.0*, *Gc=2.0*, $\lambda = 0.01, Fs = 0.01$, fw =0.2, G_1=1.0, G_2=2.0, *Pr=0.71*, *R=0.01*, *Sc* = 0.6 and $\tau = 2.0$. These values are kept as constant in entire study except the varied parameters as shown in figures 1–5 and Tables 1. The results obtained shows the influences of the non dimensional governing parameters, namely magnetic field parameter, thermal and solutal Grashof numbers, coupling parameter, Darcy parameter, vortex viscosity parameter, spin-gradient viscosity parameter, modified Forchheimer number, radiation parameter, Schmidt number and thermophoretic parameter on velocity, microratation, temperature and concentration profiles.

For different values of suction parameter fw on the velocity and microrotation profiles are presented in Figs. 1-2. It is notice that the velocity profiles increases with the increase of suction parameter fw indicating the usual fact that suction stabilizes the boundary layer growth. (See Fig. 1). The results also show that the microrotation profiles decreases near the porous plate and opposite trends far away from the porous plate as suction parameter fw increases where viscosity is dominant. Fig. 3 shows the velocity profiles for different values of modified Forchheimer number *Fs*. From this figure Fig. 3 we see that velocity decreases with the increase of *Fs*. For different values of thermal radiation parameter *R*, the temperature profiles are plotted in Fig. 4. The results show that temperature on the vertical plate increases near the porous plate and opposite trends far away from the plate as increasing radiation parameter.

The effects of various values of Schmidt number *Sc* on the concentration profiles are presented in Fig. 5. We observe that the concentration profiles decrease with increasing values of the Schmidt number *Sc.*

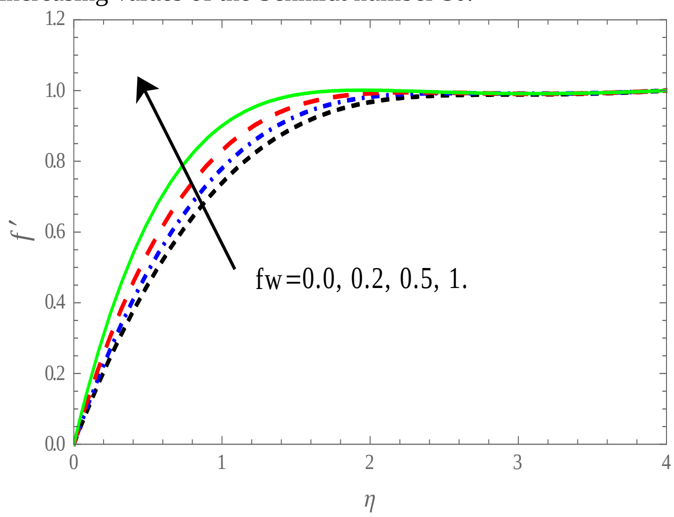

Fig 1. Effects of fw on velocity distribution $f'(\eta)$

International Journal of Engineering Research
Volume No.9, Issue No.1, pp: 05-09
ISSN: 2319-6890 (online),2347-5013(print)
1 Jan. 2020

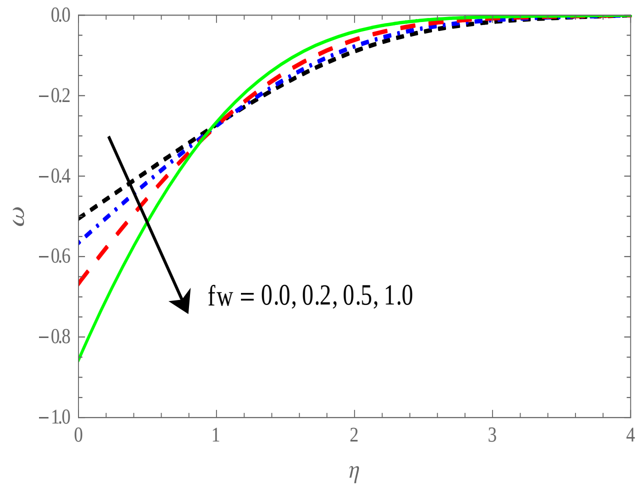

Fig 2. Effects of fw on Microrotation distribution $\omega(\eta)$

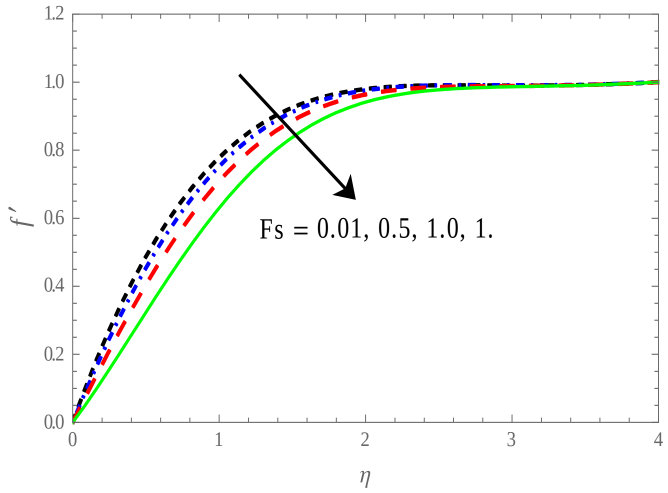

Fig 3. Effects of Fs on velocity distribution $f'(\eta)$

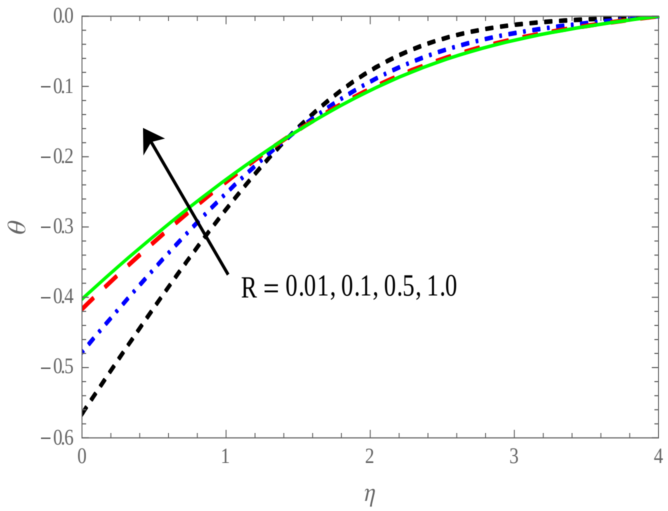

Fig 4. Effects of R on temperature distribution $\theta(\eta)$

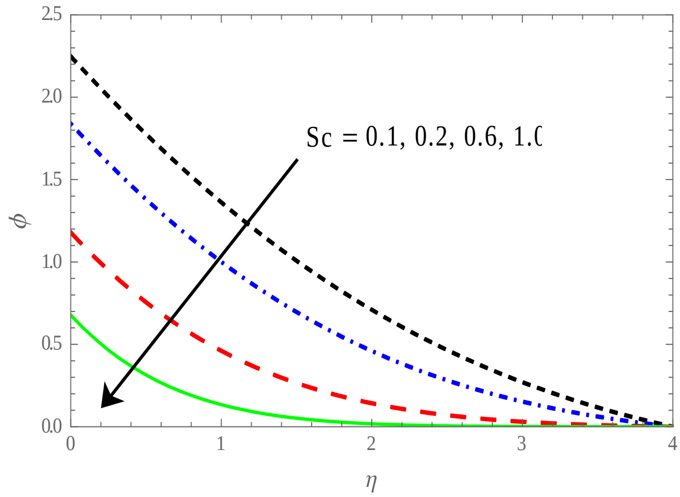

Fig 5. Effects Sc of on concentration distribution $\phi(\eta)$

The numerical values of the skin-friction coefficient, local Nusselt number and local Stanton number are tabulated in Table 1 for different values of magnetic parameter M and coupling constant K, local Darcy parameter λ, modified Forchheimer number Fs. Analysis of the tabular data Table 1 shows that magnetic parameter, modified Forchheimer number increase the values of the skin-friction coefficient, where as reverse trend is seen by increasing the values of K, λ. It is also observed that the couple stress coefficient increase with increasing values of K, λ. whereas opposite effect is seen with increasing of M, Fs. Further, it is clear that the local Nusselt number increases with increase M, Fs whereas reverse trend is seen by increasing values of K and λ. Finally, the effect of increasing the value of M, Fs has the tendency to increase local Stanton number but the other parameters like K, λ have the effect of decreasing $-\phi'(0)$.

Table 1: Numerical values of skin-friction coefficient, couple stress coefficient, local Nusselt number and local Sherwood number

M	K	λ	Fs	C_f	C_w	Nu_x	Sh_x
.1				-1.2664	0.4083	0.9246	0.7506
.2				-1.2438	0.3845	0.9286	0.7512
.5				-1.2238	0.3361	0.9237	0.7532
	.1			-1.4255	0.4242	0.7766	0.5421
	.5			-1.4660	0.5261	0.7542	0.5368
	1			-1.5319	0.6345	0.7364	0.5214
		.1		-1.1224	0.4083	0.9846	0.8992
		.5		-1.1282	0.4495	0.9840	0.8981
		1		-1.1341	0.4930	0.9834	0.8980
			.1	-1.4158	0.3936	0.7812	0.5419
			.5	-1.4074	0.3248	0.7874	0.5439
			1	-1.3936	0.2277	0.7974	0.5472

VI. Conclusions

- The temperature increases near the surface and then decreases way from the surface with increase in *R*.
- The concentration decreases with increase in Schmidt number *Sc*.

- Both Nu_x and Sh_x increases with an increase in *K*, λ and R and they increases with in *M* and *Fs.*

Acknowledgement

The authors are thankful to the editor and reviewers for their continuous support in the process of publishing this article.

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